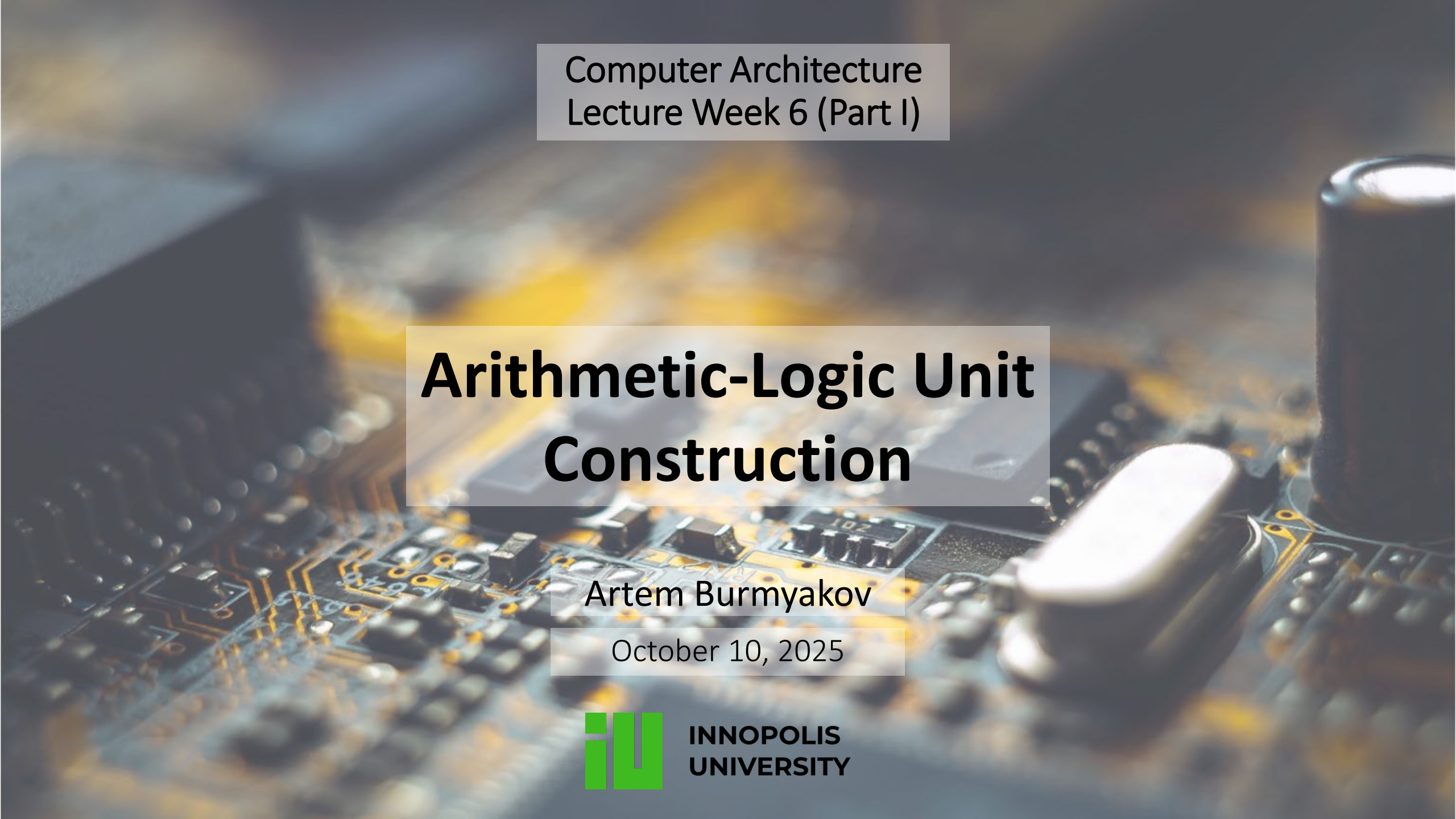




Computer Architecture
Lecture Week 6 (Part I)

Arithmetic-Logic Unit Construction

Artem Burmyakov

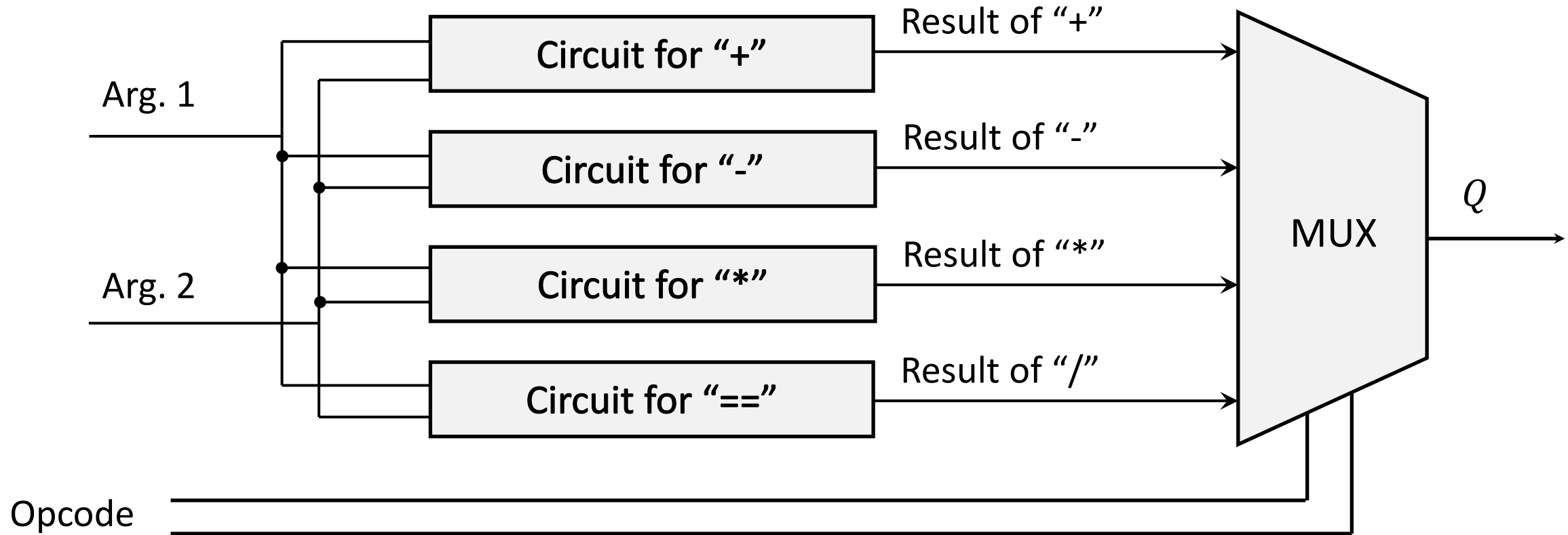
October 10, 2025



INNOPOLIS
UNIVERSITY

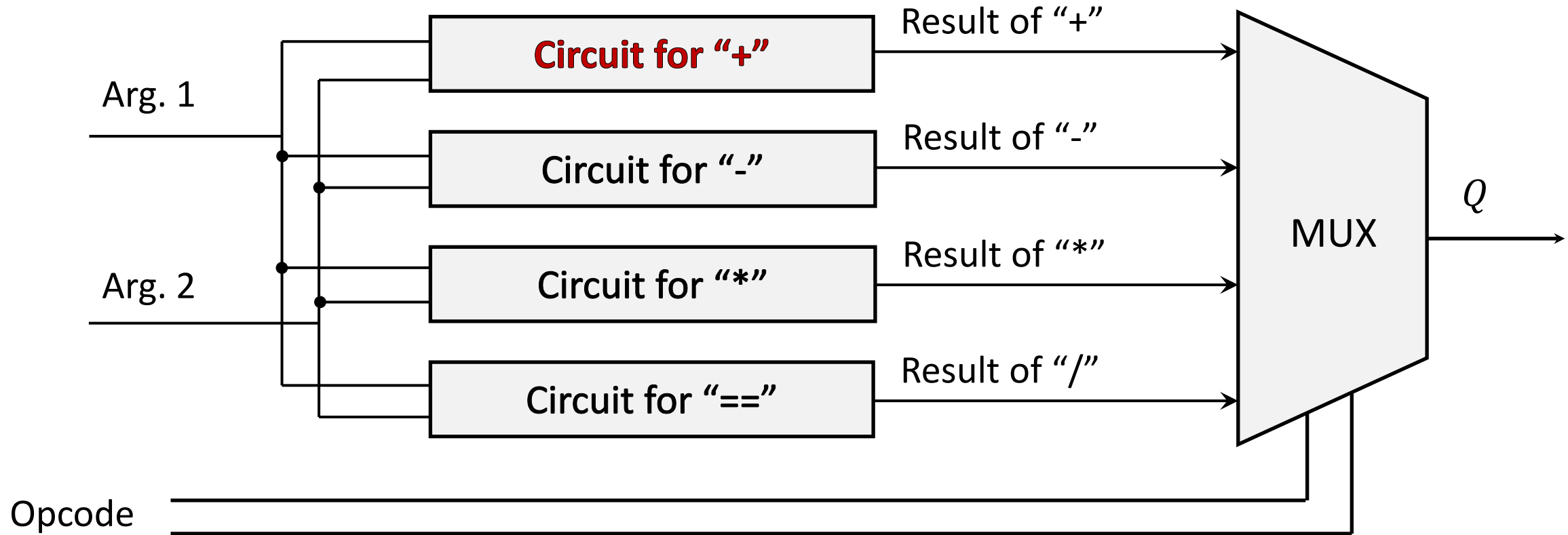
Arithmetic Logic Unit (ALU): Basic Organization

Case of 4 instructions: "+", "-", "*", "/"

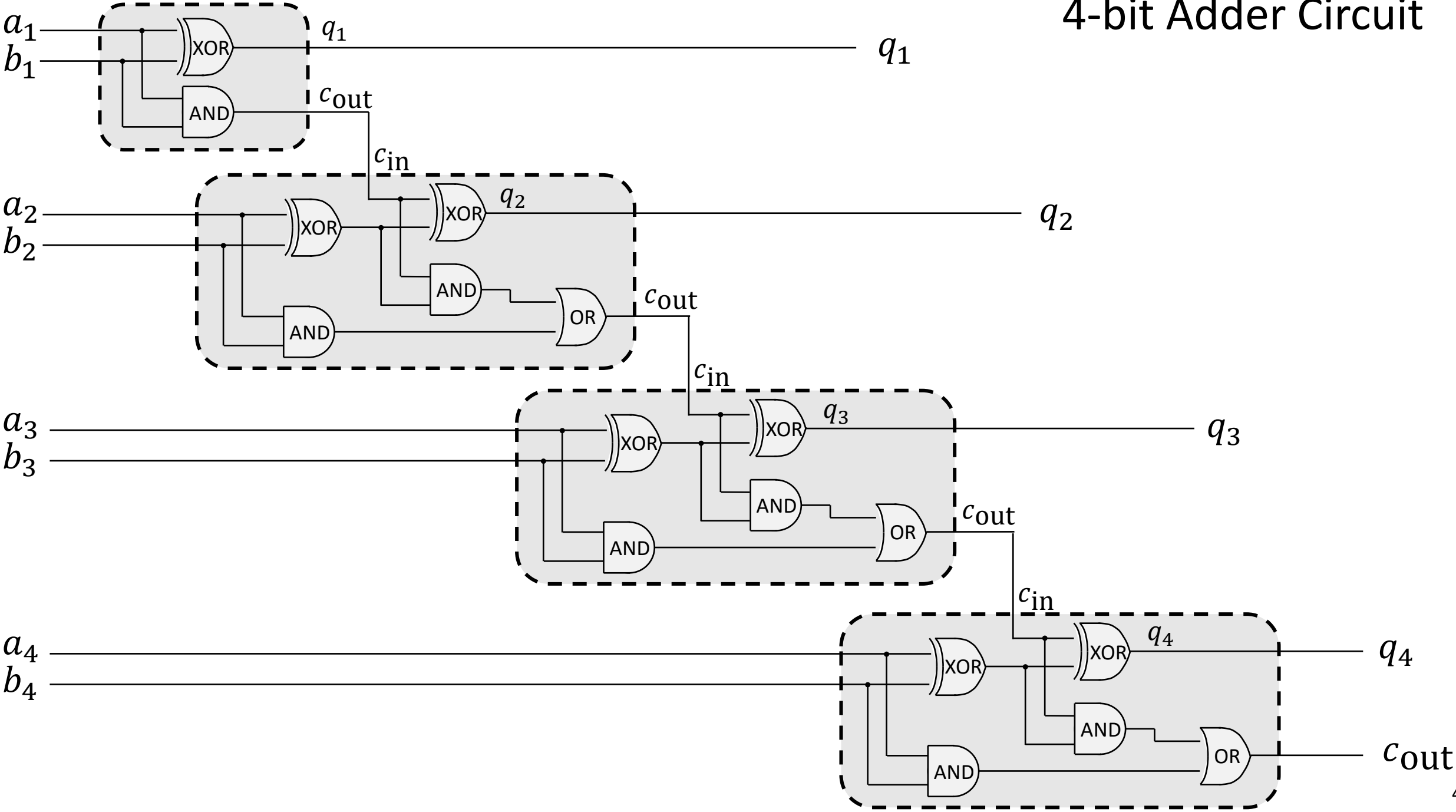


Arithmetic Logic Unit (ALU): Basic Organization

Case of 4 instructions: "+", "-", "*", "/"

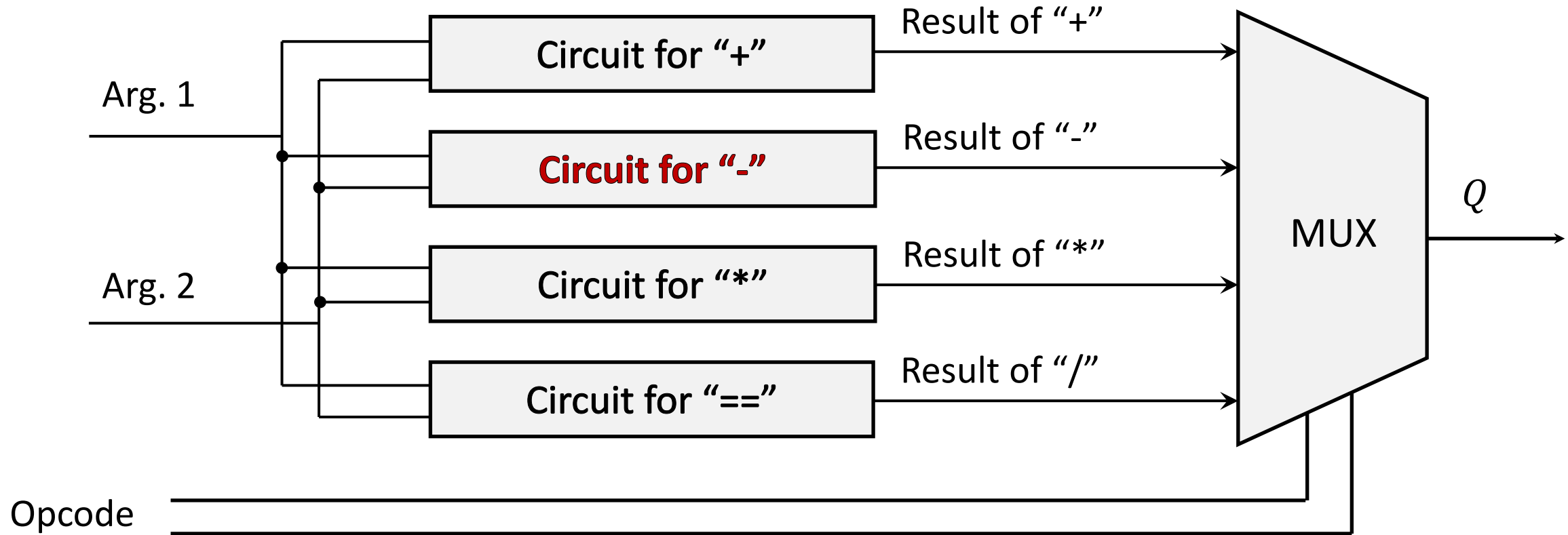


4-bit Adder Circuit

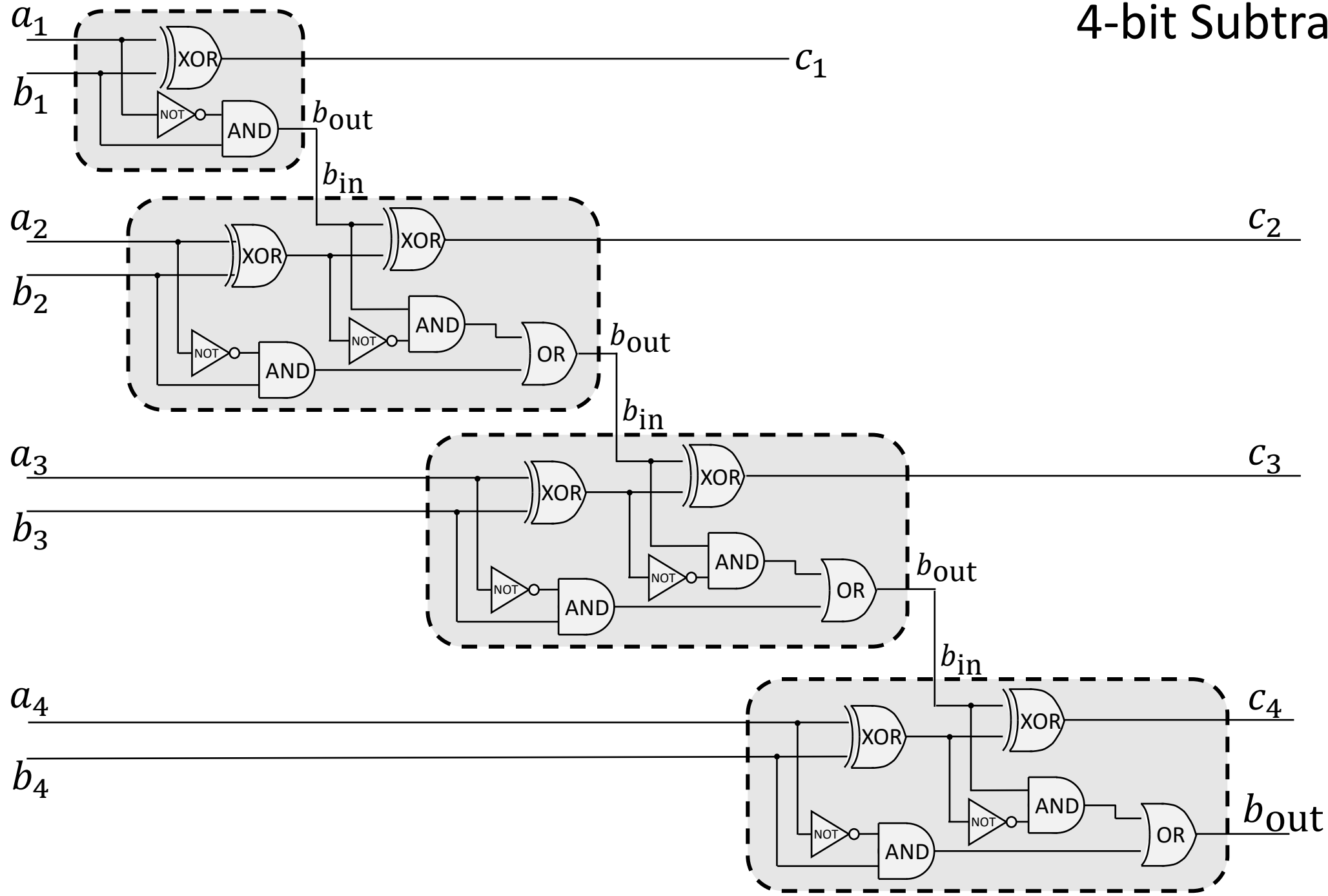


Arithmetic Logic Unit (ALU): Basic Organization

Case of 4 instructions: "+", "-", "*", "/"

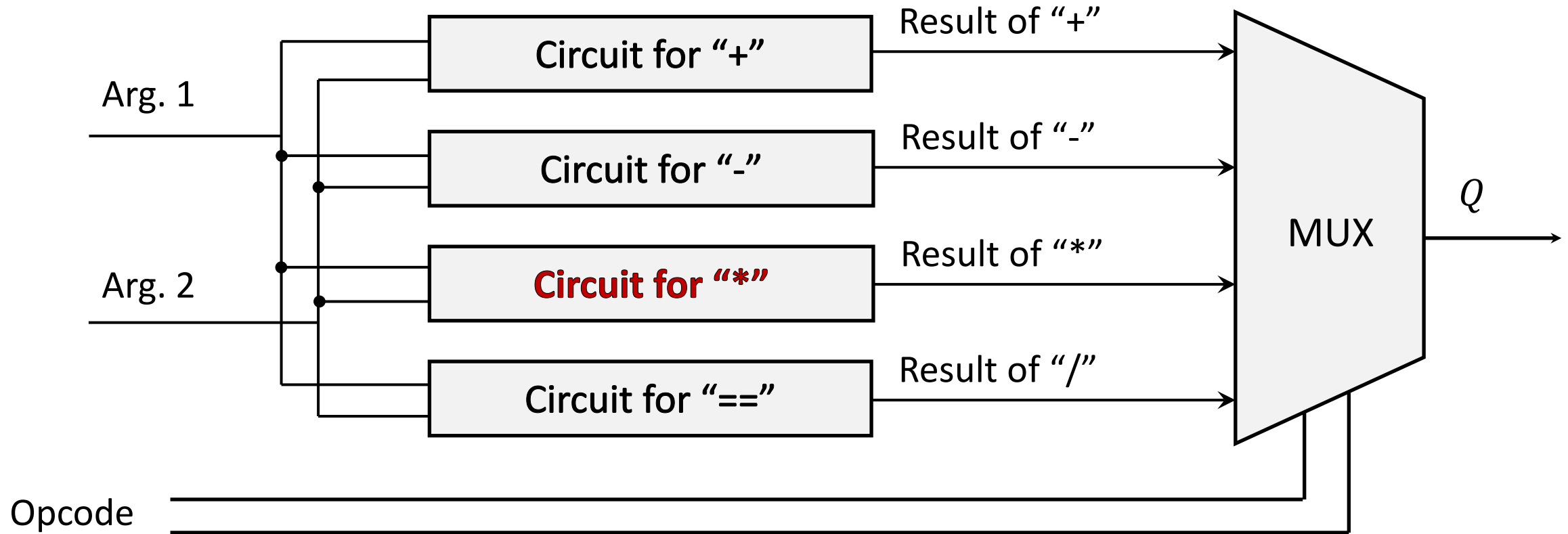


4-bit Subtractor Circuit



Arithmetic Logic Unit (ALU): Basic Organization

Case of 4 instructions: "+", "-", "*", "=="



Binary Multipliers (positive arguments)

Single bits case:

$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$

Binary Multipliers (positive arguments)

Single bits case:

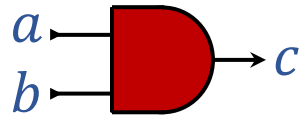
$$\underline{a \times b = c}$$

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Binary Multipliers (positive arguments)

Single bits case:

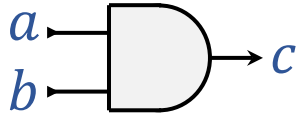
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Binary Multipliers (positive arguments)

Single bits case:

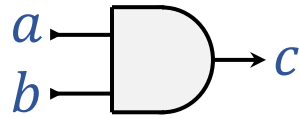
$$\underline{a \times b = c}$$

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2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$\mathbf{a_1 a_0} \times 1 = \mathbf{a_1 a_0}$$

$$a_1 a_0 \times \mathbf{0} = \mathbf{00}$$

Binary Multipliers (positive arguments)

Single bits case:

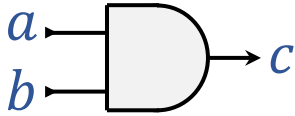
$$\underline{a \times b = c}$$

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$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

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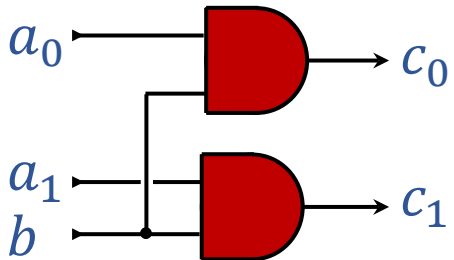
2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Every bit of $a_2 a_1$ is multiplied
by b :



Binary Multipliers (positive arguments)

Single bits case:

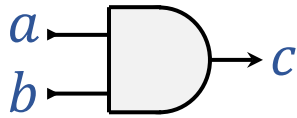
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

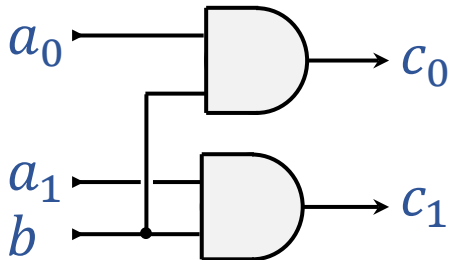
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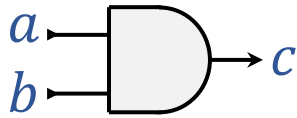
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$\mathbf{11} \times \mathbf{11} = \mathbf{1001} \quad \text{4 bits -- max. result bitsize}$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

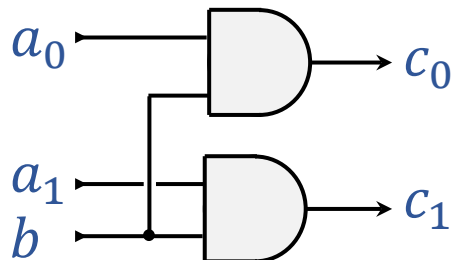
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied
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Binary Multipliers (positive arguments)

Single bits case:

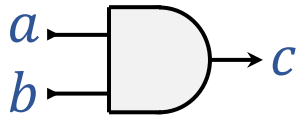
$$a \times b = c$$

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$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001 \text{ 4 bits -- max. result bitsize}$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

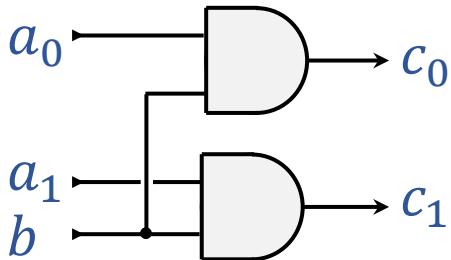
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Binary Multipliers (positive arguments)

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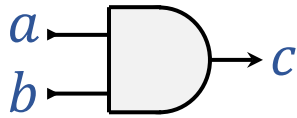
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$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

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$$00 \times 11 = 00$$

...

×

	a_1	a_0
	b_1	b_0

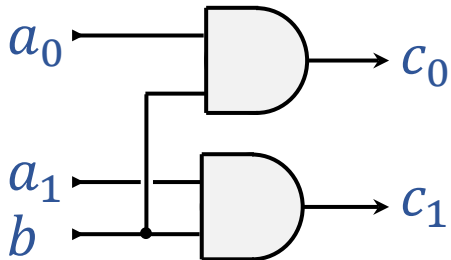
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Binary Multipliers (positive arguments)

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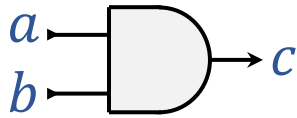
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$$00 \times 11 = 00$$

...

×	a_1	a_0
	b_1	b_0
	a_1b_0	a_0b_0

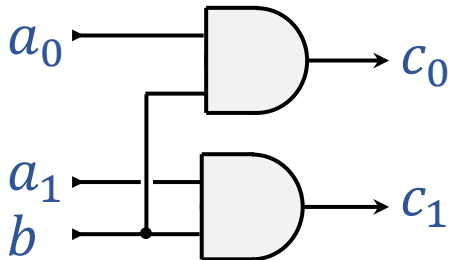
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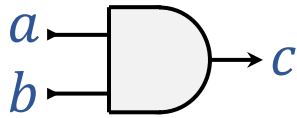
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2-by-2 bits case:

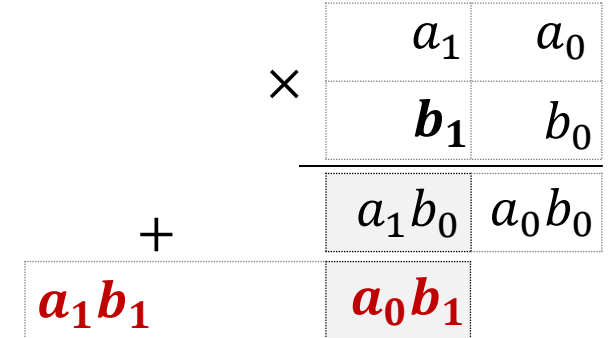
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

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...



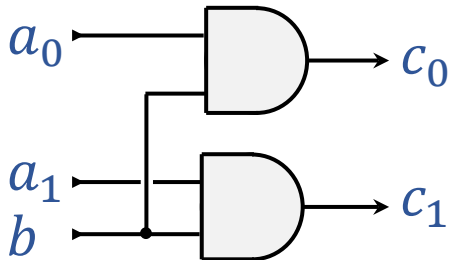
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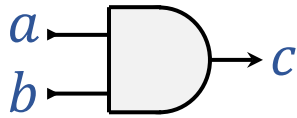
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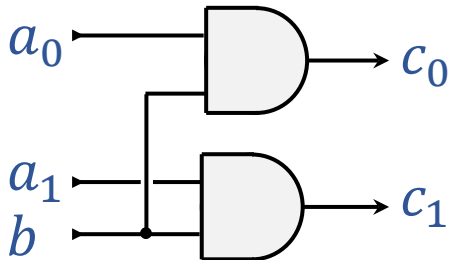
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2-by-2 bits case:

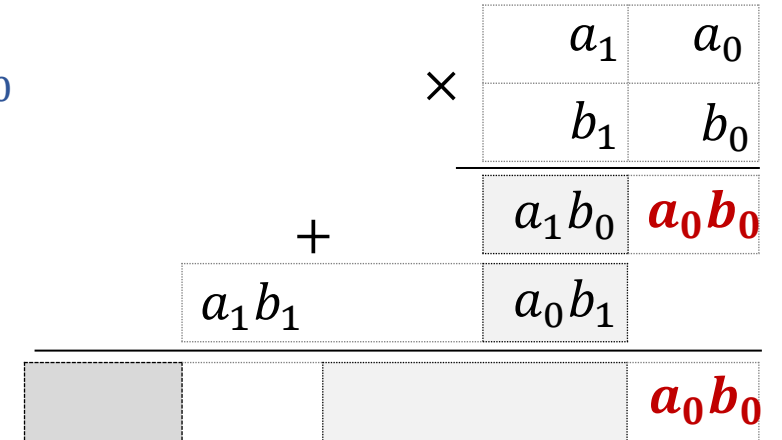
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

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...



Binary Multipliers (positive arguments)

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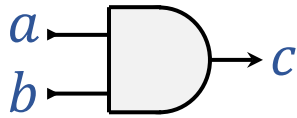
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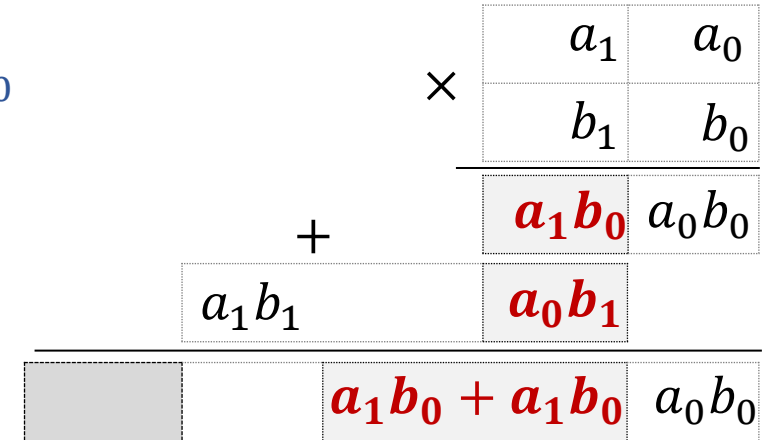
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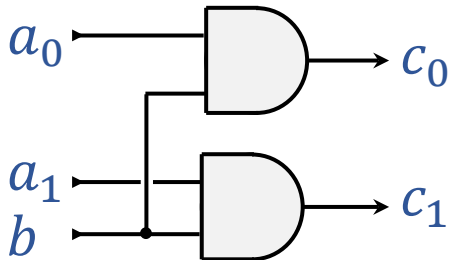
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Binary Multipliers (positive arguments)

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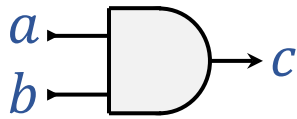
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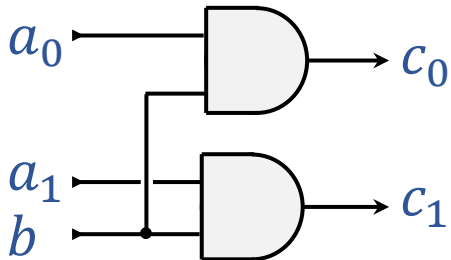
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2-by-2 bits case:

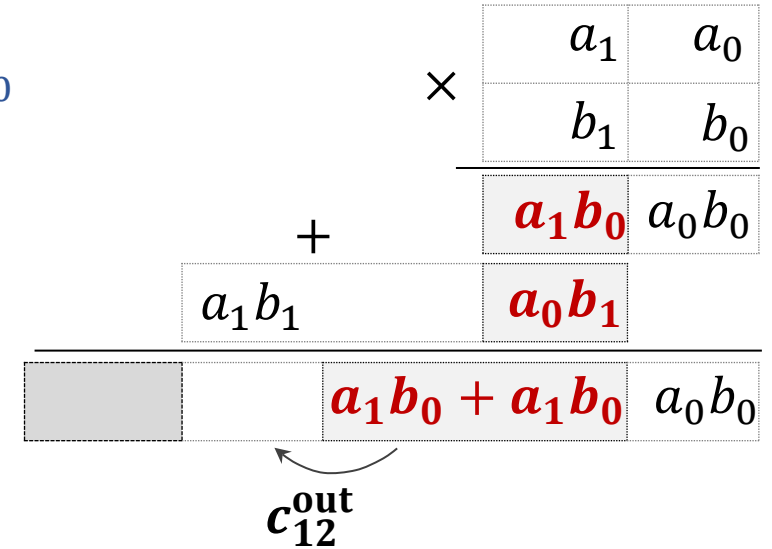
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

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...



Binary Multipliers (positive arguments)

Single bits case:

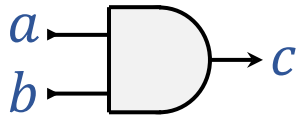
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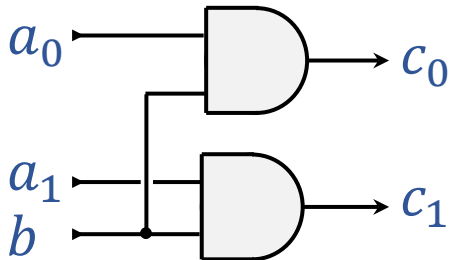
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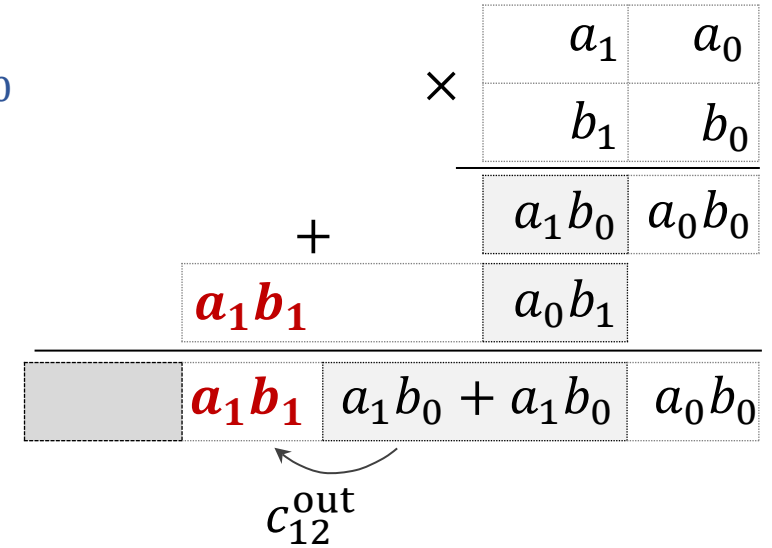
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Binary Multipliers (positive arguments)

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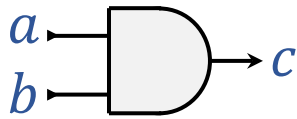
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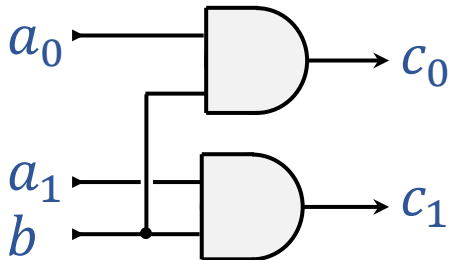
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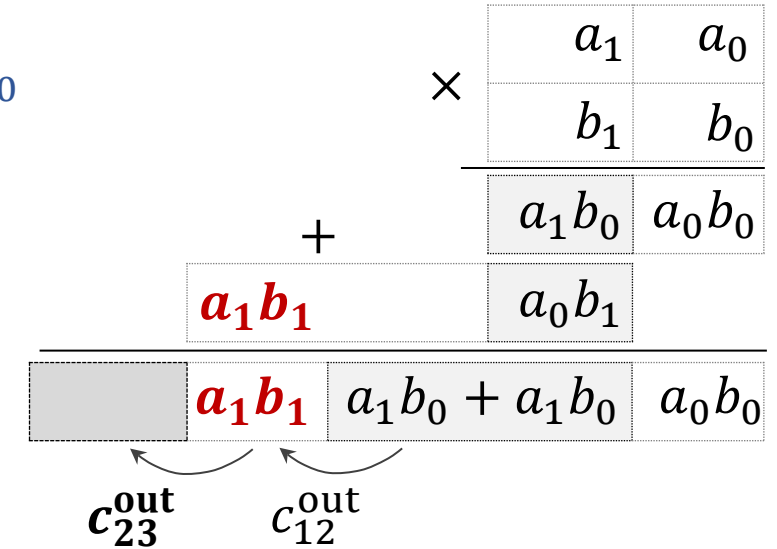
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Binary Multipliers (positive arguments)

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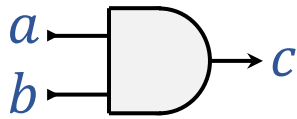
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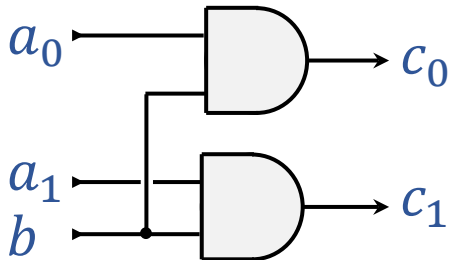
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2-by-2 bits case:

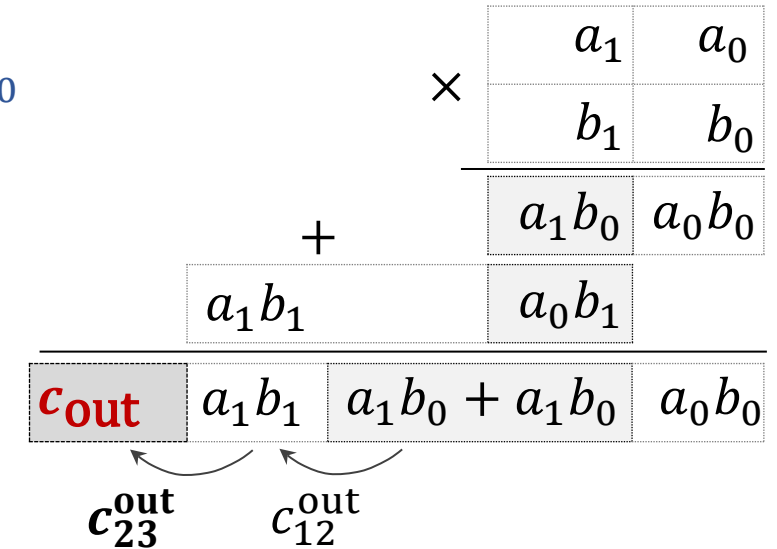
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Binary Multipliers (positive arguments)

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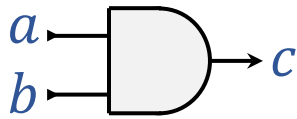
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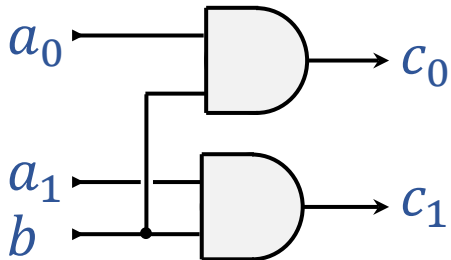
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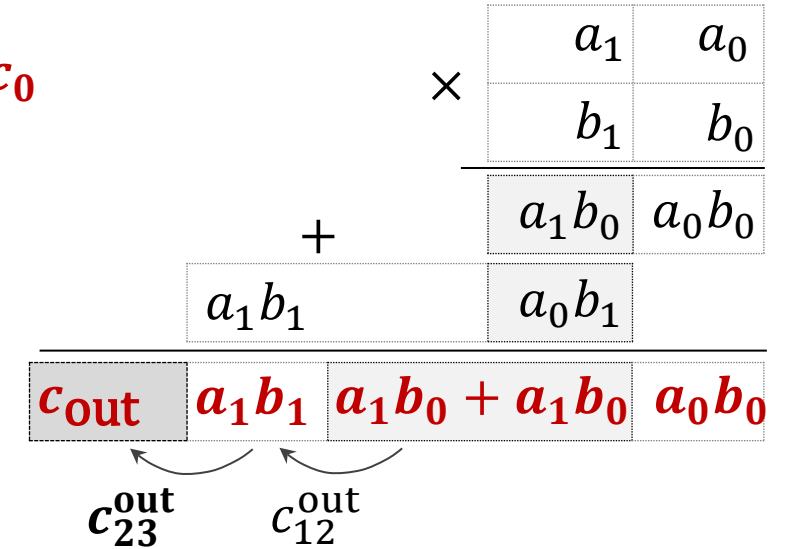
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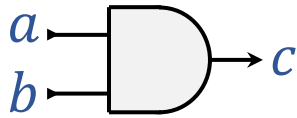
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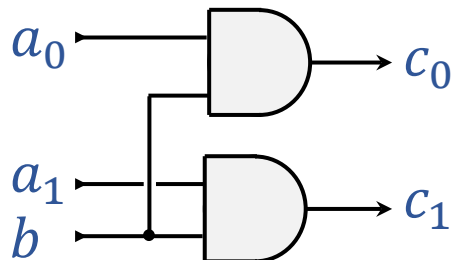
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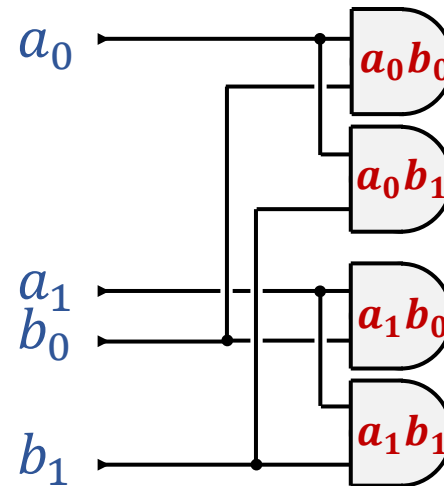
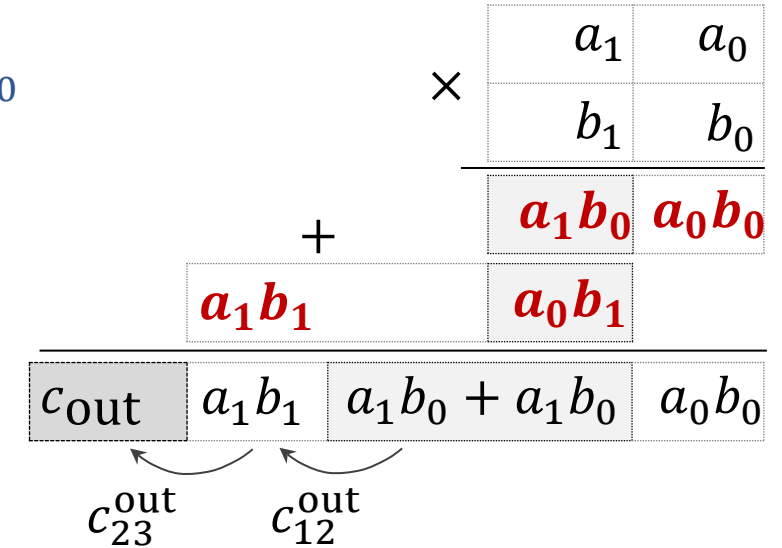
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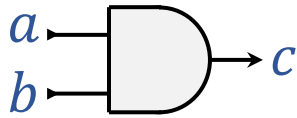
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$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



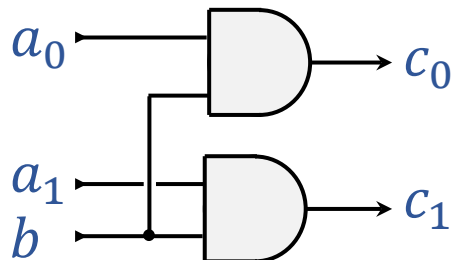
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

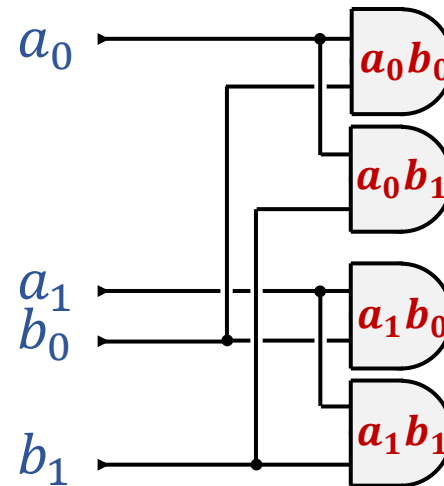
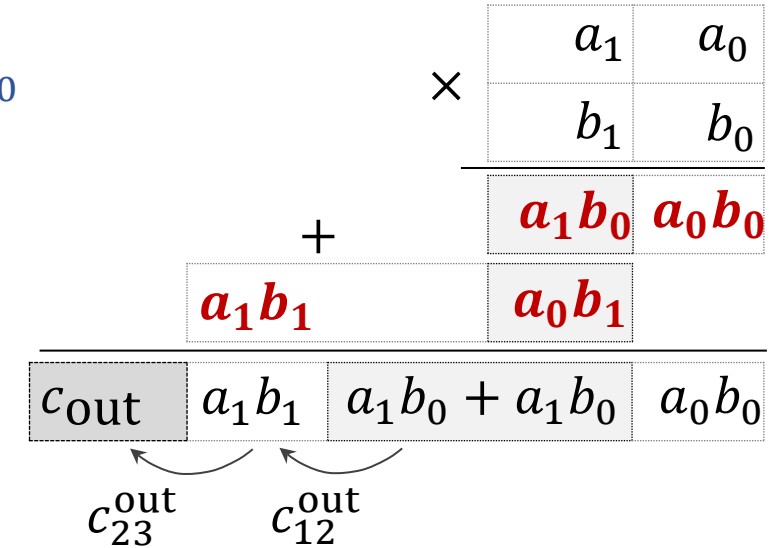
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



<- Partial products

Binary Multipliers (positive arguments)

Single bits case:

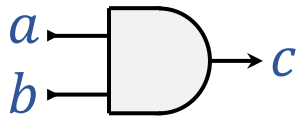
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



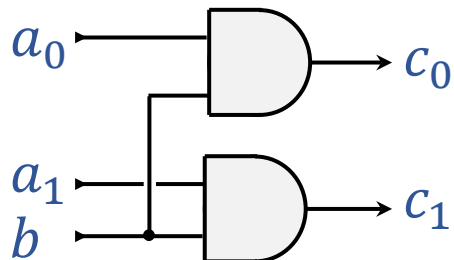
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

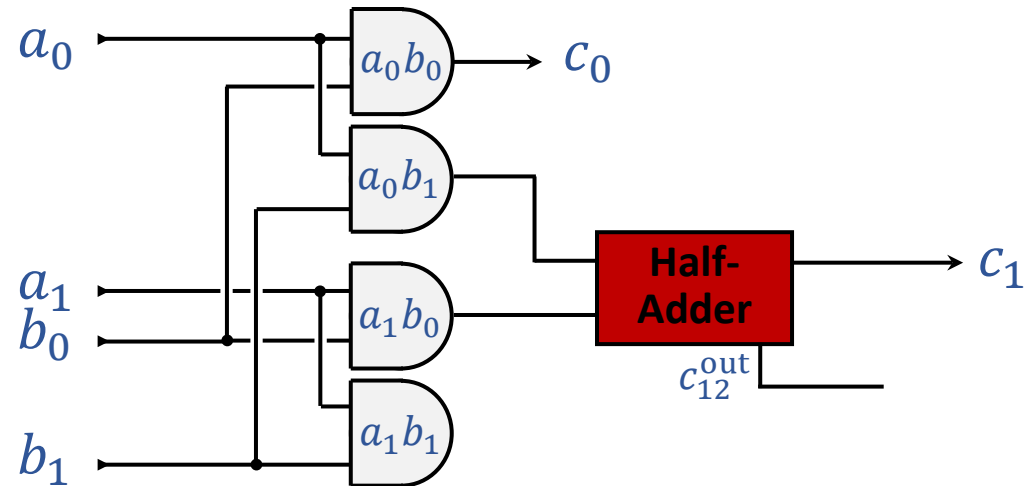
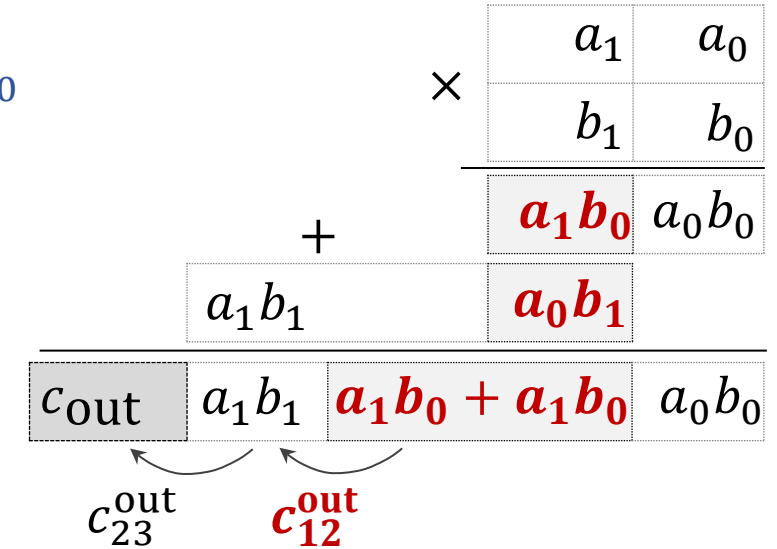
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers (positive arguments)

Single bits case:

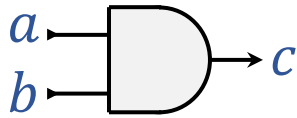
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



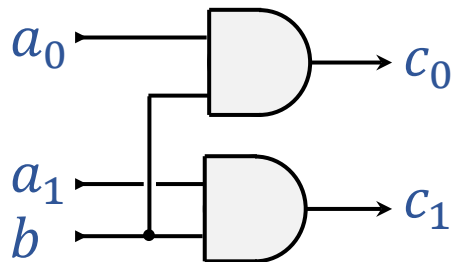
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

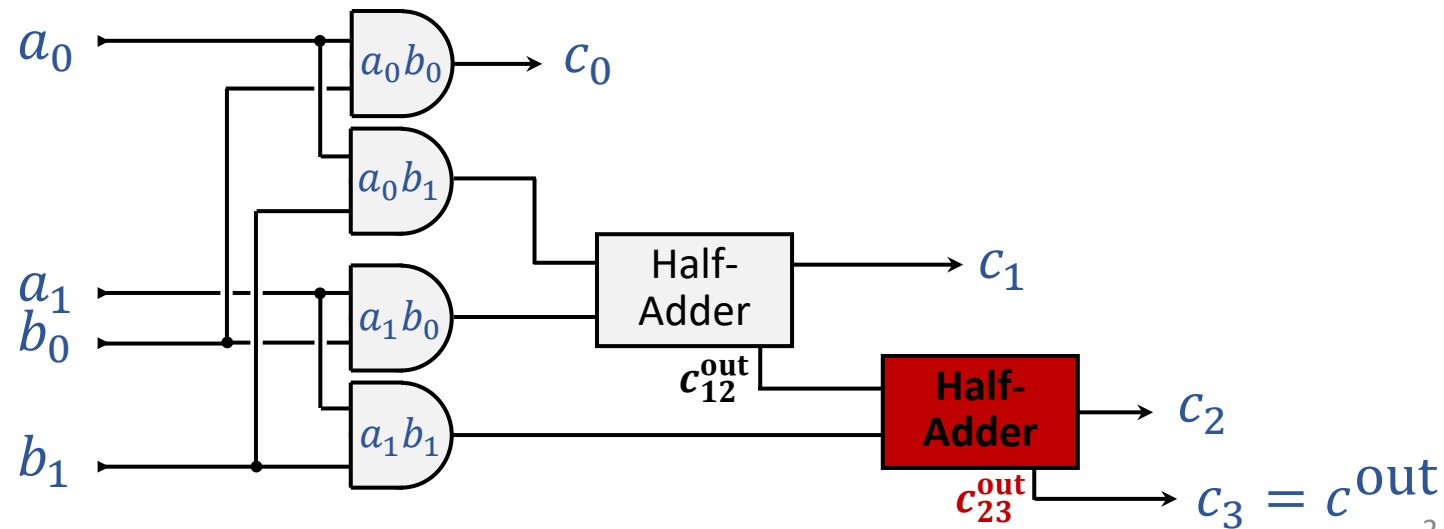
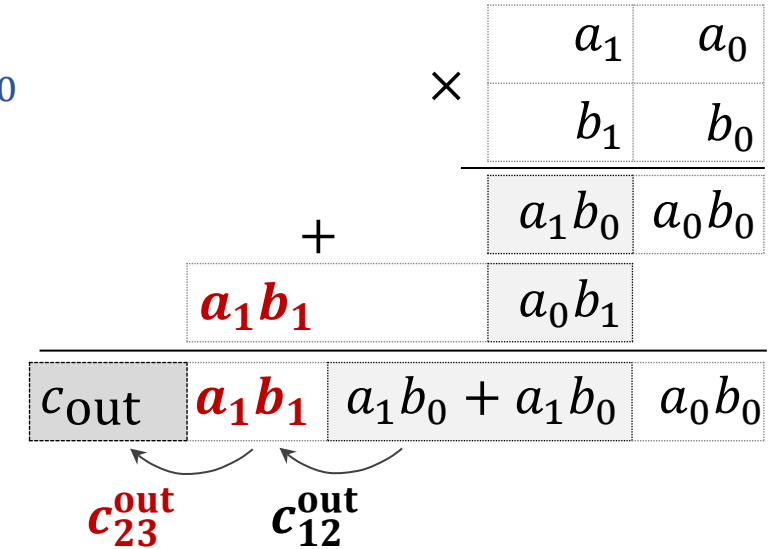
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

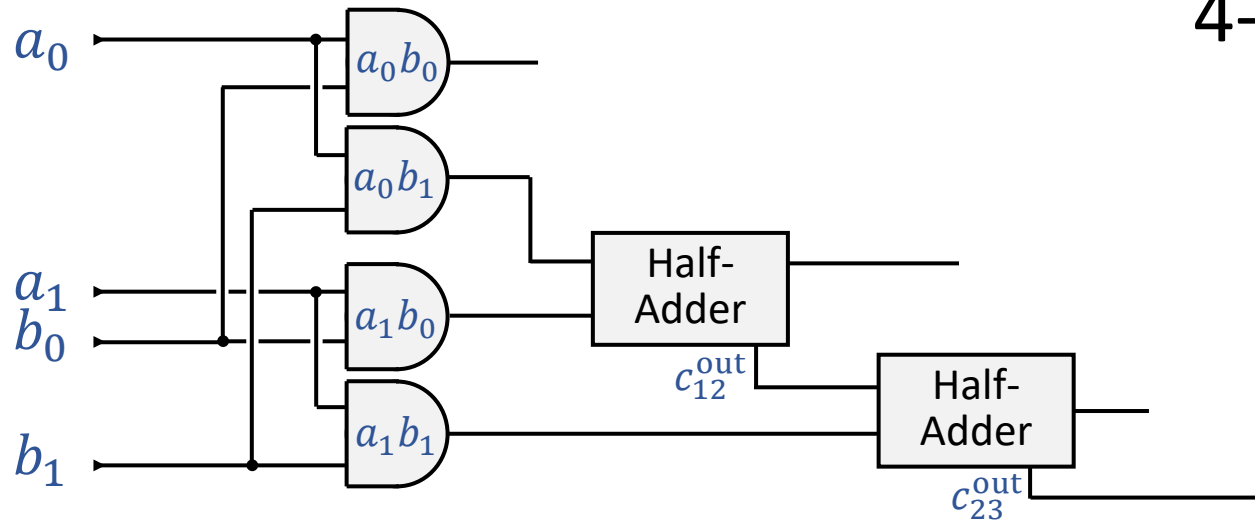
$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

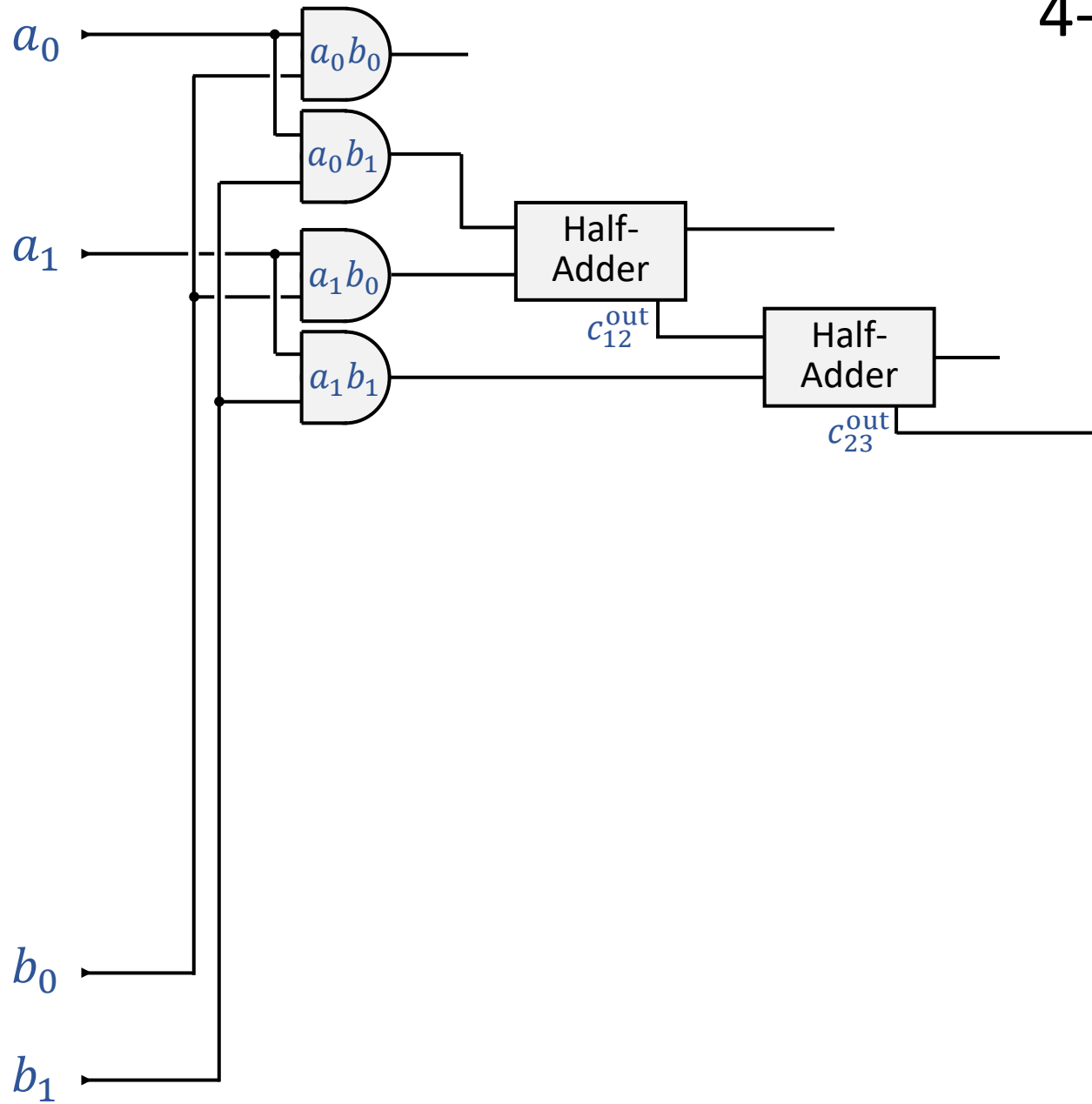


4-by-2 Bit Multiplier Construction



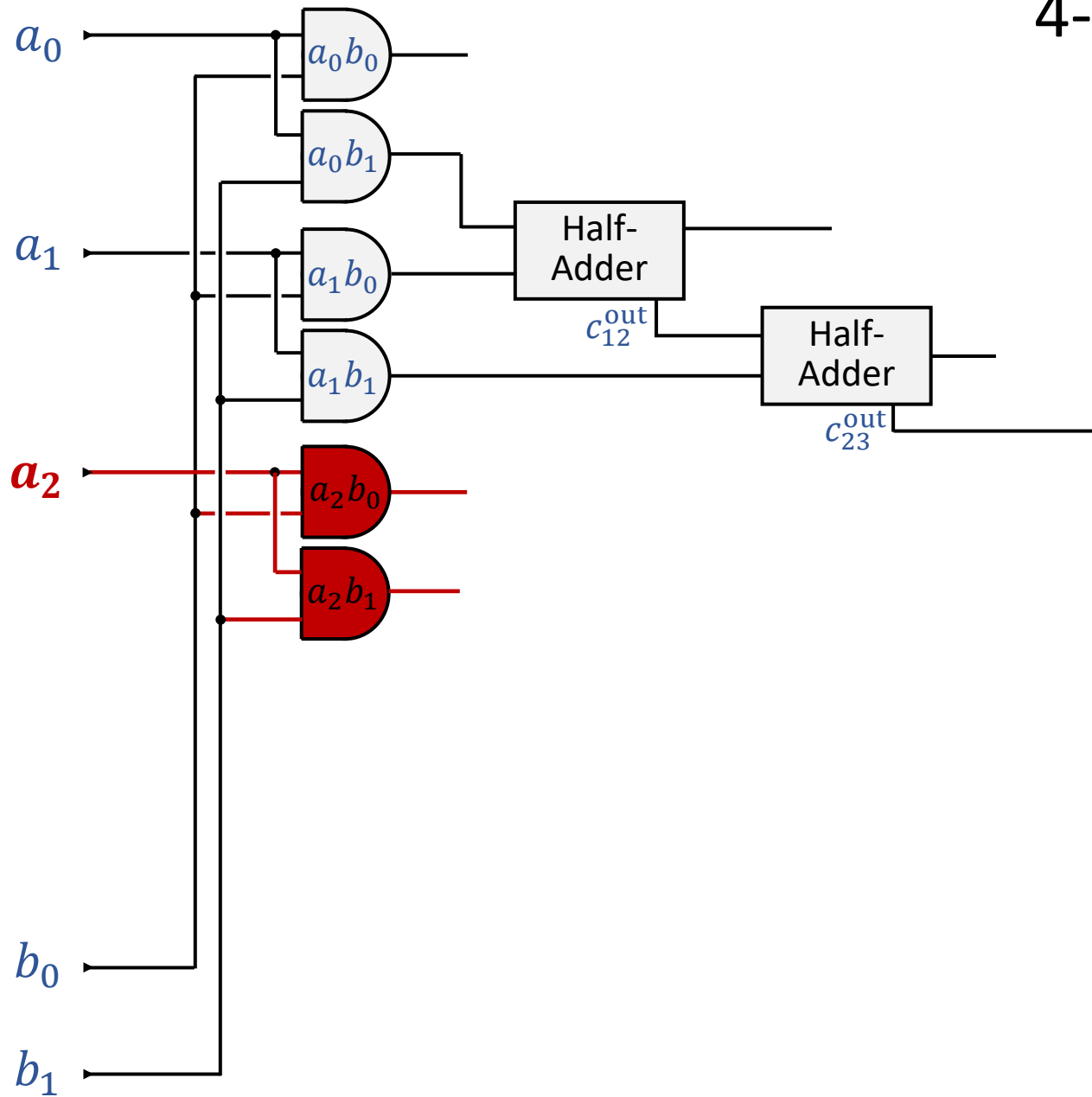
$$\begin{array}{r} \times \begin{array}{|c|c|} \hline a_1 & a_0 \\ \hline b_1 & b_0 \\ \hline \end{array} \\ + \begin{array}{|c|c|} \hline a_1b_0 & a_0b_0 \\ \hline a_1b_1 & a_0b_1 \\ \hline \end{array} \\ \hline \end{array}$$

4-by-2 Bit Multiplier Construction



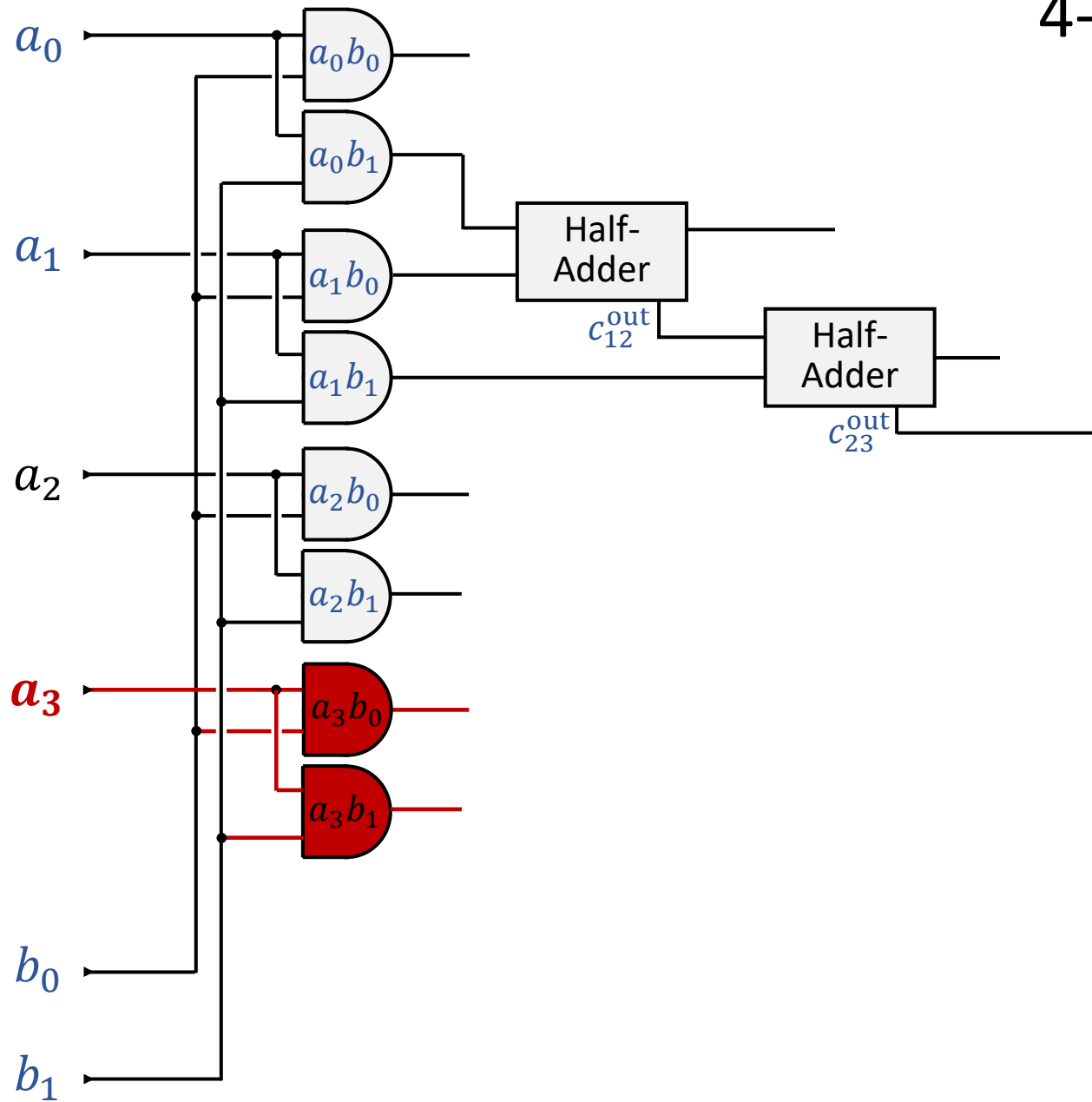
$$\begin{array}{r} \times \begin{array}{|c|c|} \hline a_1 & a_0 \\ \hline b_1 & b_0 \\ \hline \end{array} \\ + \begin{array}{|c|c|} \hline a_1b_0 & a_0b_0 \\ \hline a_1b_1 & a_0b_1 \\ \hline \end{array} \end{array}$$

4-by-2 Bit Multiplier Construction



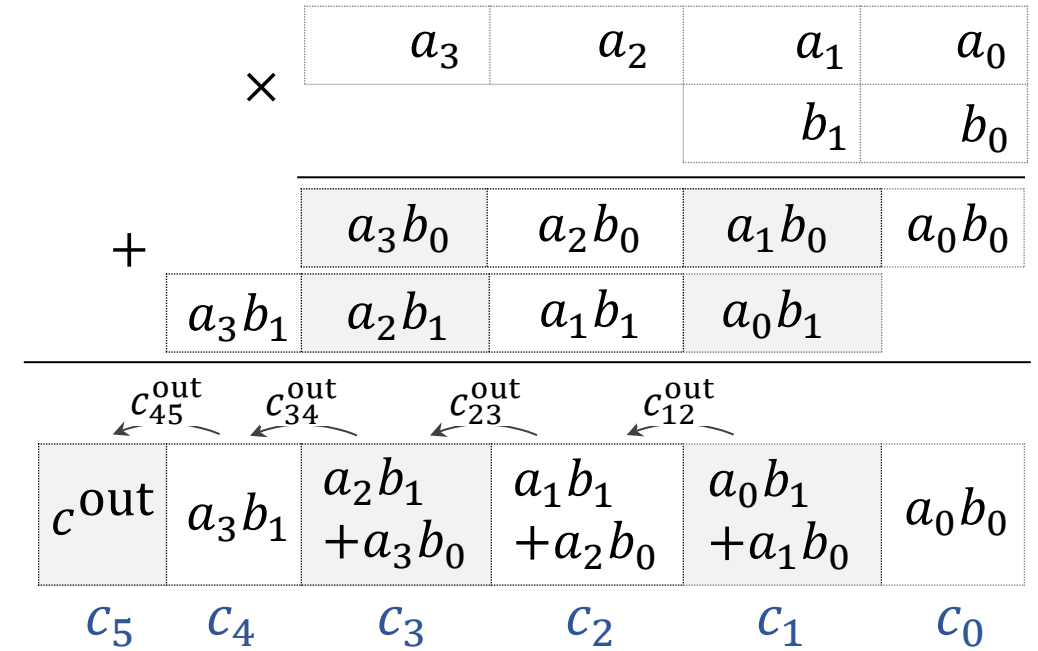
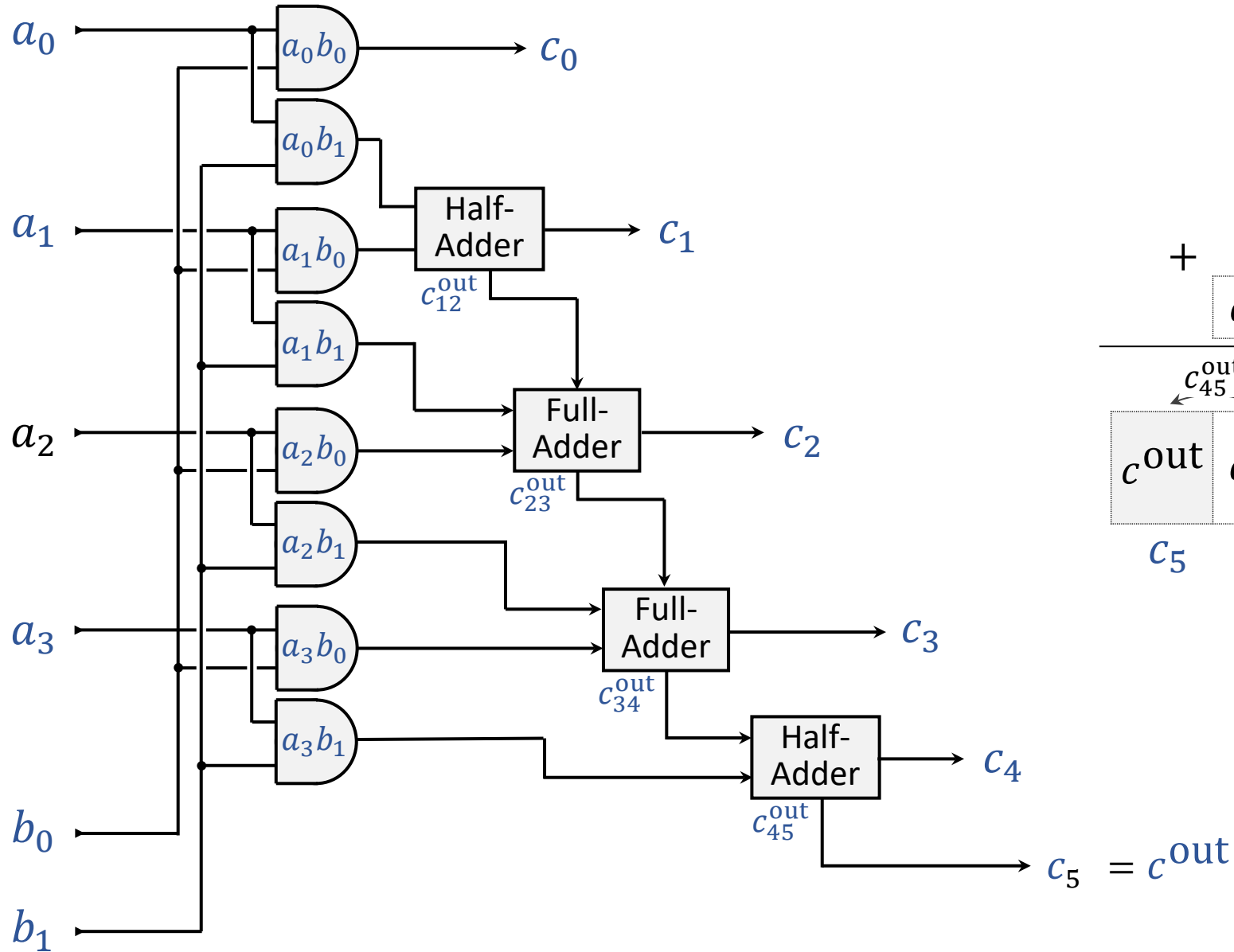
$$\begin{array}{r} \times \begin{array}{|c|c|c|} \hline a_2 & a_1 & a_0 \\ \hline & b_1 & b_0 \\ \hline \end{array} \\ + \begin{array}{|c|c|c|} \hline a_2b_0 & a_1b_0 & a_0b_0 \\ \hline a_2b_1 & a_1b_1 & a_0b_1 \\ \hline \end{array} \end{array}$$

4-by-2 Bit Multiplier Construction

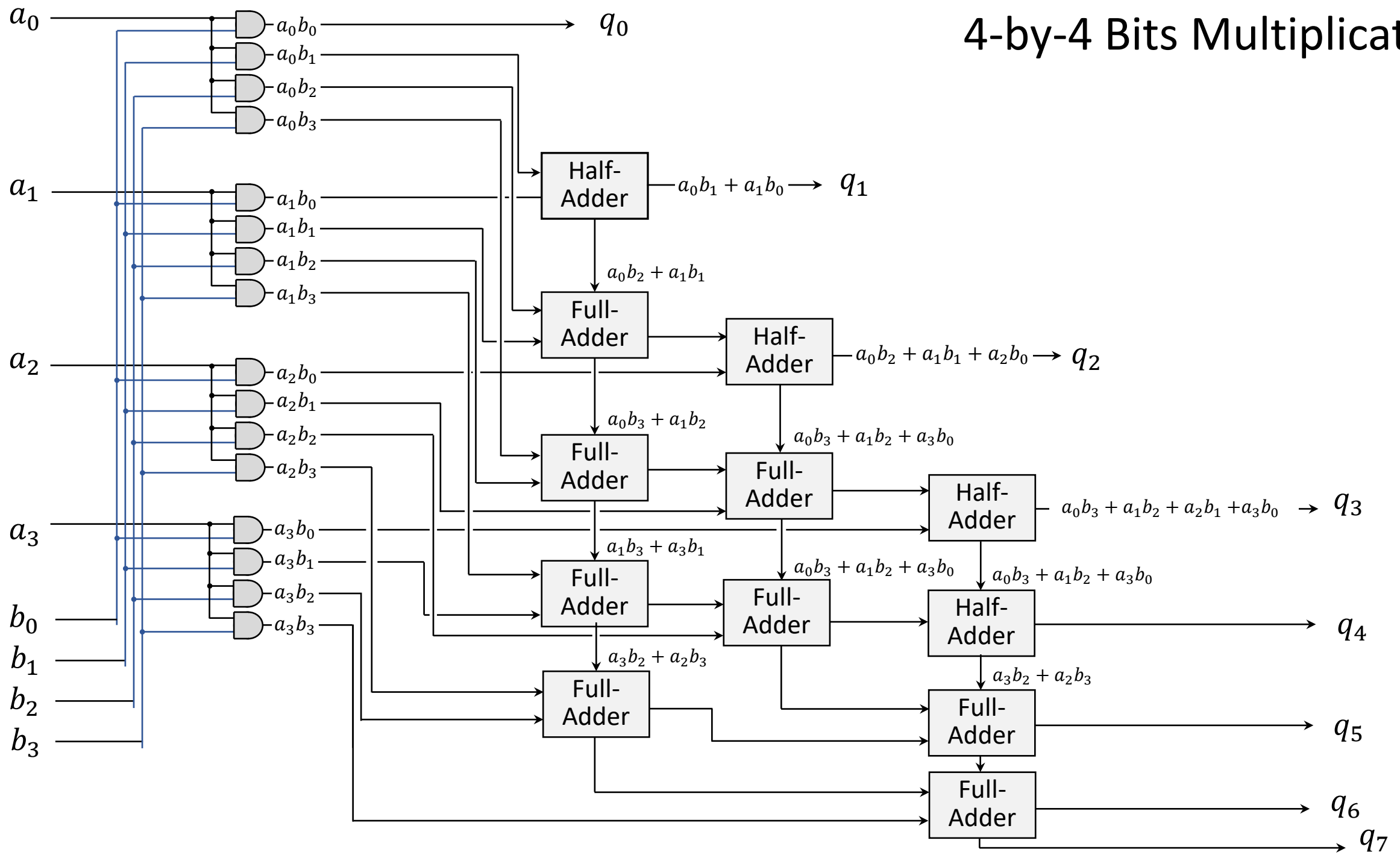


×	a_3	a_2	a_1	a_0
			b_1	b_0
+	$a_3 b_0$	$a_2 b_0$	$a_1 b_0$	$a_0 b_0$
	$a_3 b_1$	$a_2 b_1$	$a_1 b_1$	$a_0 b_1$

4-by-2 Bit Multiplier Construction

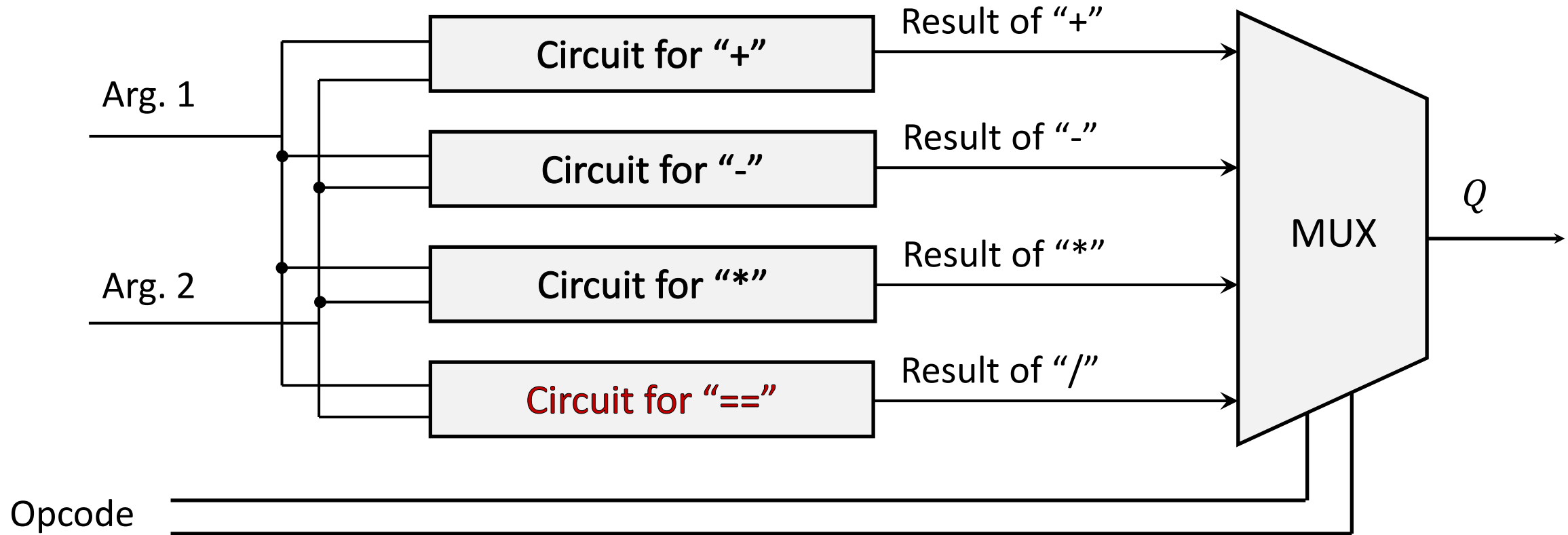


4-by-4 Bits Multiplier



Arithmetic Logic Unit (ALU): Basic Organization

Case of 4 instructions: "+", "-", "*", "=="



4-by-4 Bits Equality Checker ("==" Operator)

