

Computer Architecture
Lecture Week 5 (Part II)

Equality Checker Binary Multiplier

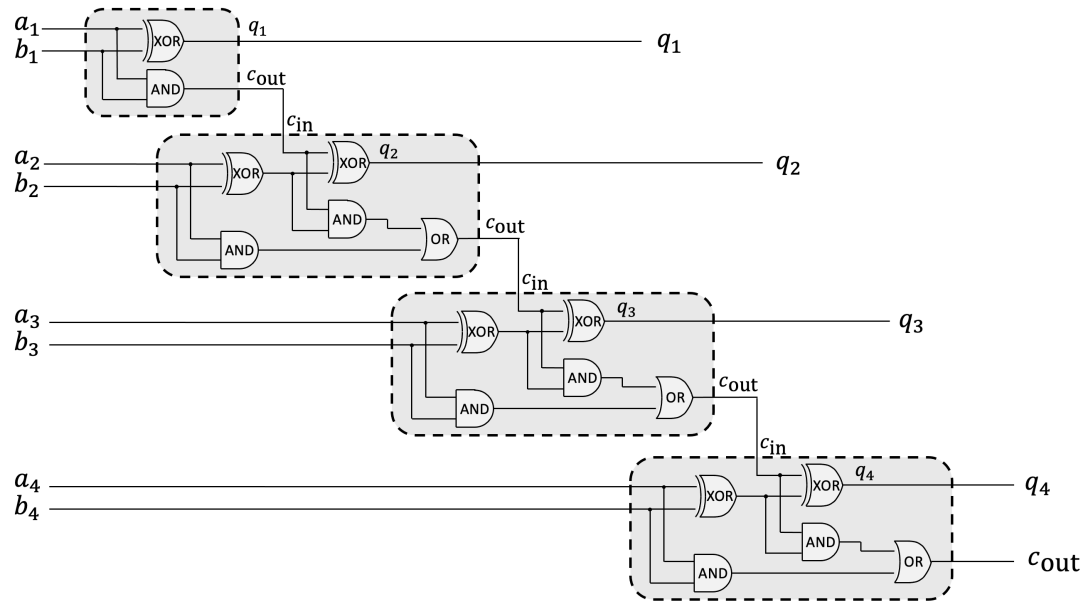
Artem Burmyakov

October 02, 2026



INNOPOLIS
UNIVERSITY

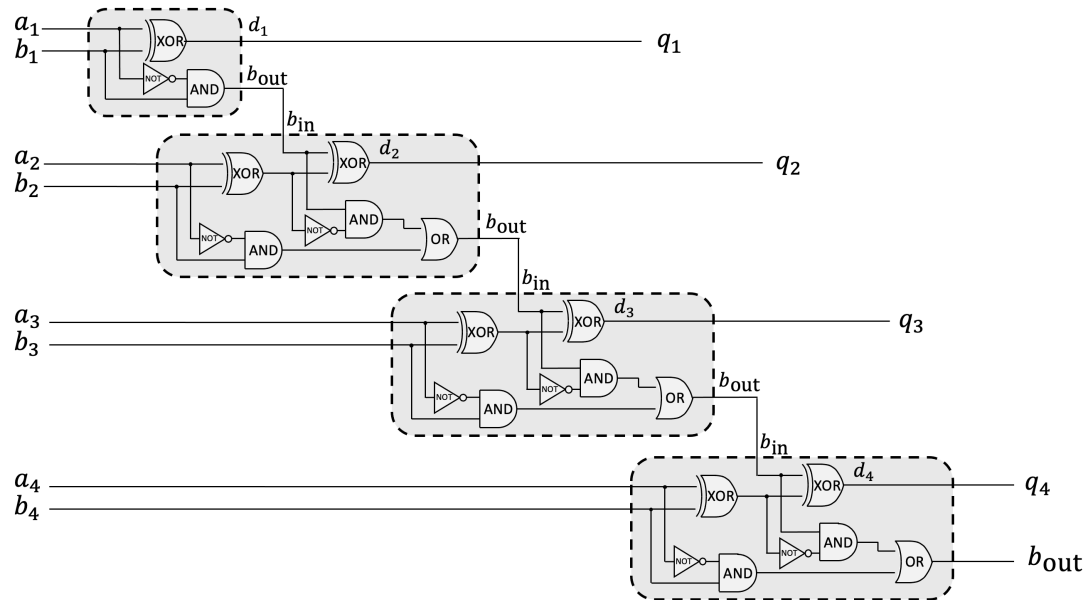
Adder



Recap: Adder and Subtractor Circuits

(4 bit input arguments)

Subtractor



Equality Checker Construction ($a \stackrel{?}{=} b$)

a_1
 b_1

Equality Checker Construction ($a \neq b$)

a_1 _____
 b_1 _____

Equality Checker Construction ($a=b$)

a_1 _____
 b_1 _____

XOR Truth Table:

a	b	q
0	0	0
0	1	1
1	0	1
1	1	0

Equality Checker Construction ($a=b$)

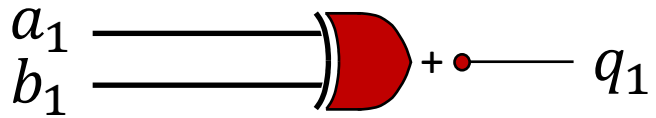
a_1 _____
 b_1 _____

XOR Truth Table:

a	b	q
0	0	0
0	1	1
1	0	1
1	1	0

**Inverted equality checker
for 1 bit arguments**

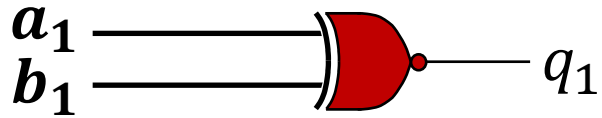
Equality Checker Construction ($a=b$)



XOR + **NOT** Truth Table:

a1	b1	q1
0	0	1
0	1	0
1	0	0
1	1	1

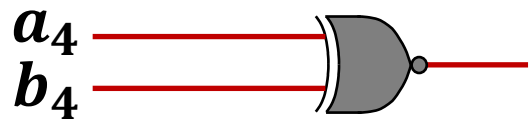
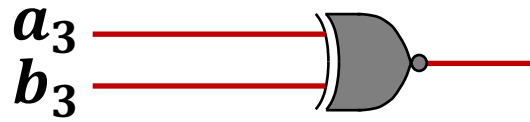
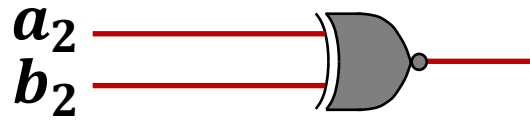
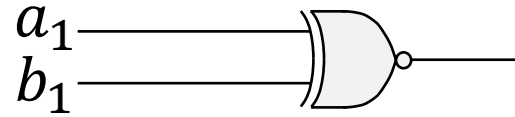
Equality Checker Construction ($a=b$)



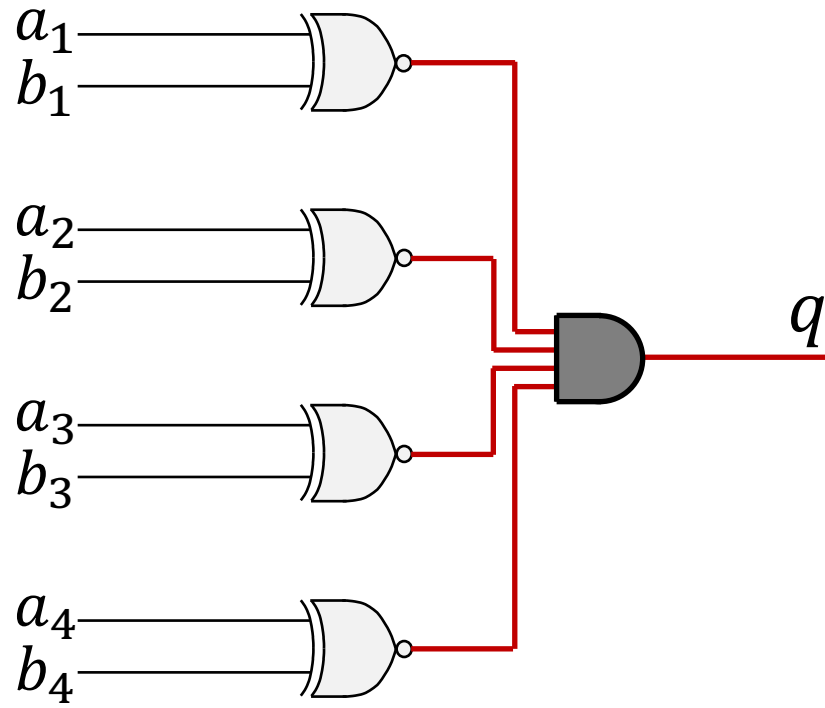
XOR + NOT Truth Table:

a1	b1	q1
0	0	1
0	1	0
1	0	0
1	1	1

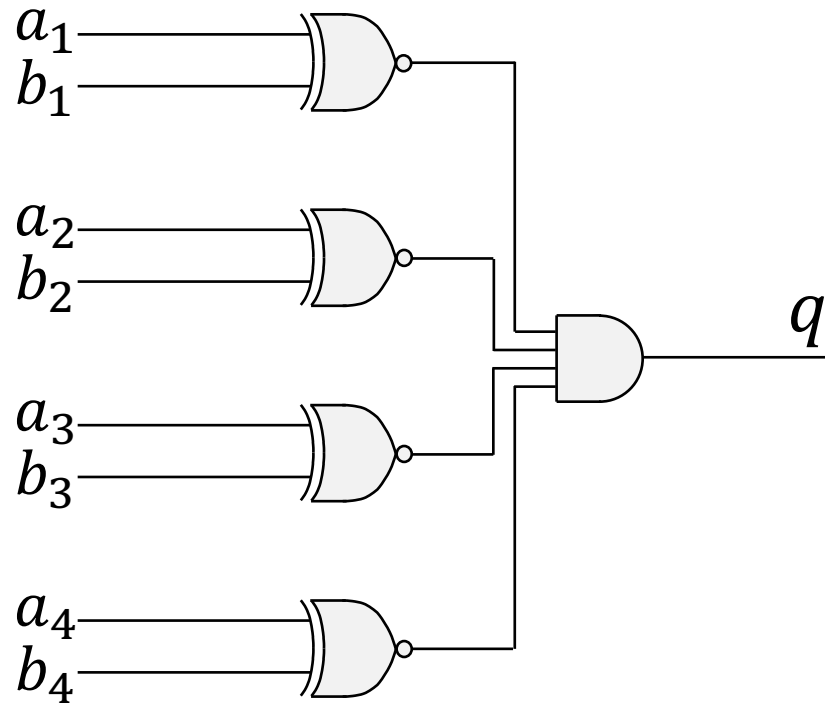
Equality Checker Construction ($a?=b$)



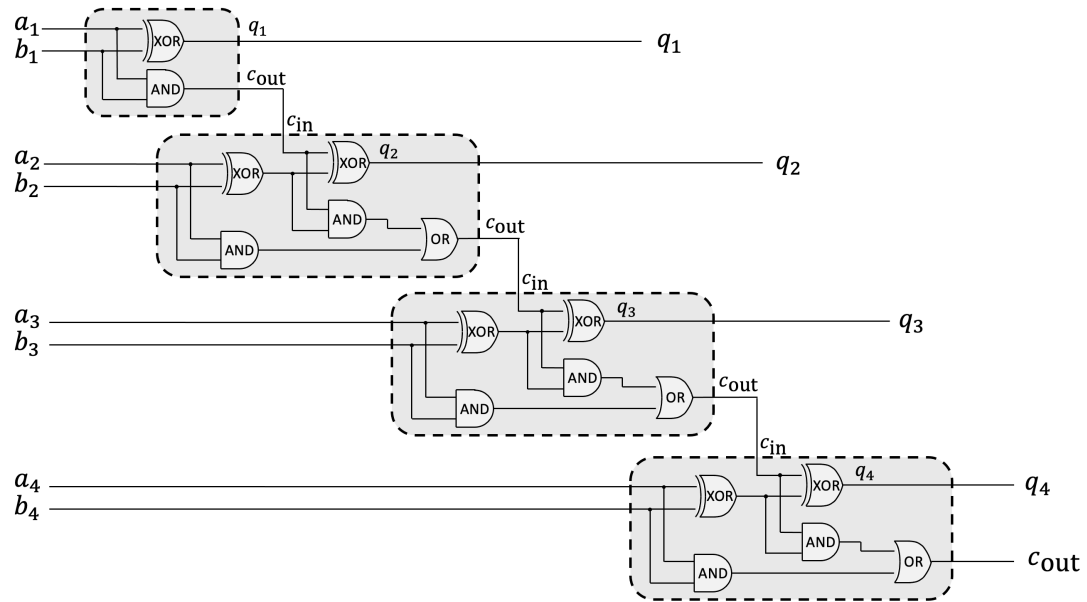
Equality Checker Construction ($a?=b$)



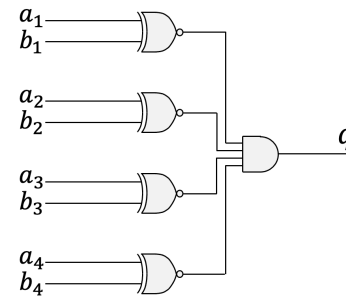
Equality Checker Construction ($a?=b$)



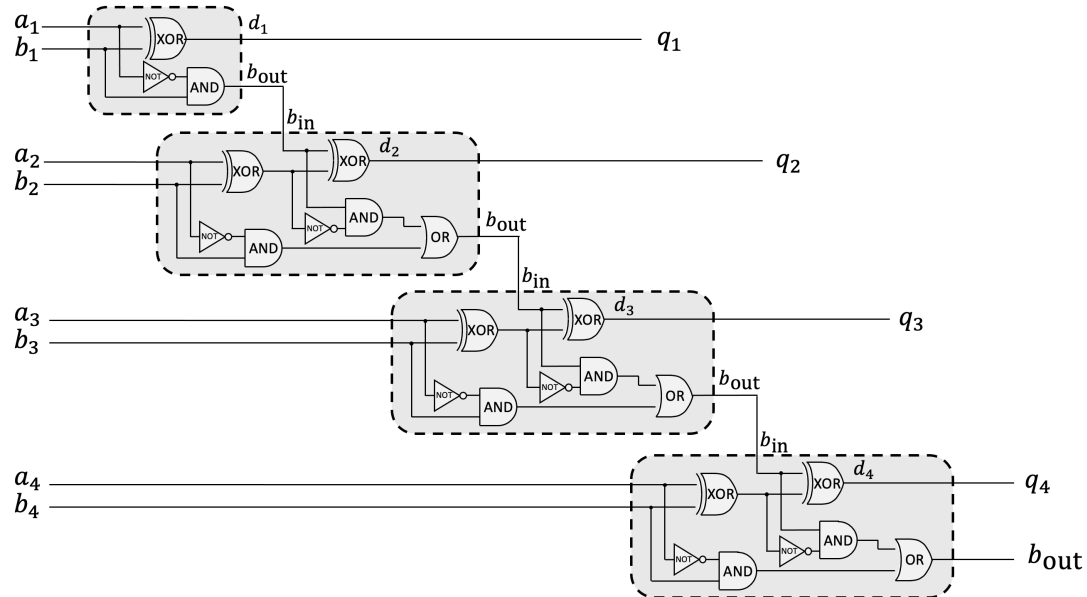
Adder (+)



Equality Checker (=?=)



Subtractor (-)



Some Instructions Implementations

Binary Multipliers (positive arguments)

1-by-1 bit case:

$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$

Binary Multipliers (positive arguments)

1-by-1 bit case:

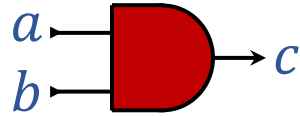
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



Binary Multipliers (positive arguments)

1-by-1 bit case:

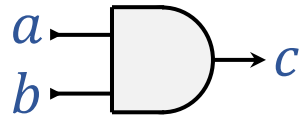
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Binary Multipliers (positive arguments)

1-by-1 bit case:

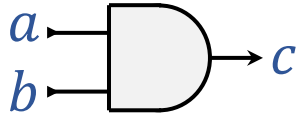
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



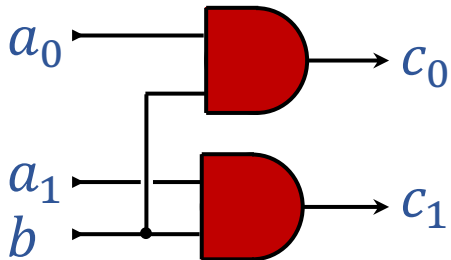
2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Every bit of $a_1 a_0$ is multiplied
by b :



Binary Multipliers

1-by-1 bit case:

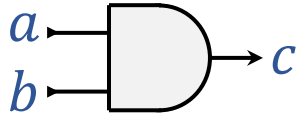
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

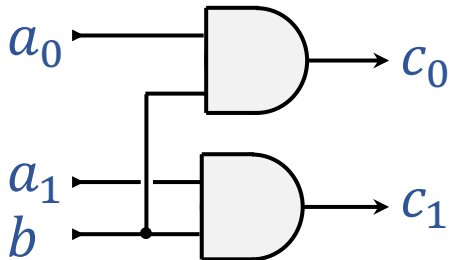
2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Every bit of $a_2 a_1$ is multiplied
by b :



Binary Multipliers

1-by-1 bit case:

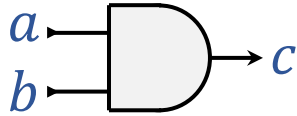
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$\mathbf{11 \times 11 = 1001} \quad \text{4 bits – max. result bitsize}$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

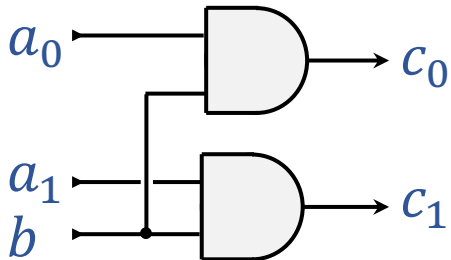
2-by-1 bits case:

$$\underline{a_1 a_0 \times b = c_1 c_0}$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Every bit of $a_2 a_1$ is multiplied
by b :



Binary Multipliers

1-by-1 bit case:

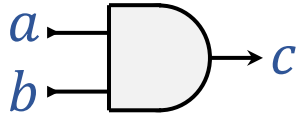
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$\underline{a_1a_0 \times b_1b_0 = c_3c_2c_1c_0}$$

$$11 \times 11 = 1001 \quad \text{4 bits – max. result bitsize}$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

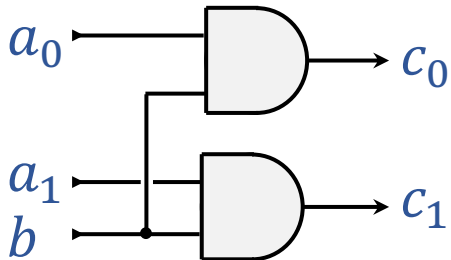
2-by-1 bits case:

$$\underline{a_1a_0 \times b = c_1c_0}$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied
by b :



Binary Multipliers

1-by-1 bit case:

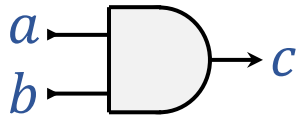
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$\underline{a_1a_0 \times b_1b_0 = c_3c_2c_1c_0}$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

×

	a_1	a_0
	b_1	b_0

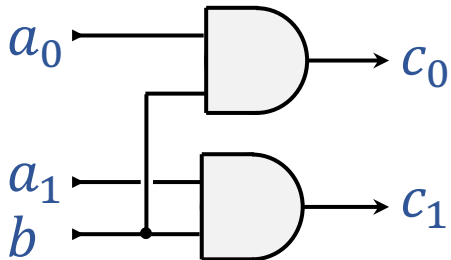
2-by-1 bits case:

$$\underline{a_1a_0 \times b = c_1c_0}$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



Binary Multipliers

1-by-1 bit case:

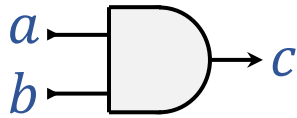
$$\underline{a \times b = c}$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

$$\underline{a_1a_0 \times b_1b_0 = c_3c_2c_1c_0}$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...

×	a_1	a_0
	b_1	b_0
	a_1b_0	a_0b_0

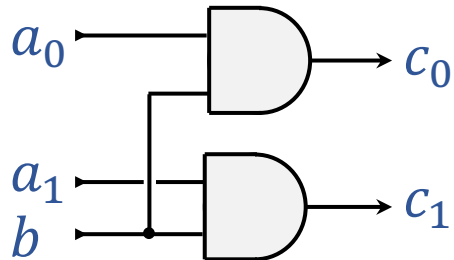
2-by-1 bits case:

$$\underline{a_1a_0 \times b = c_1c_0}$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



Binary Multipliers

1-by-1 bit case:

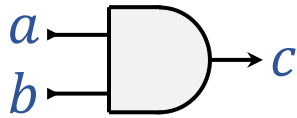
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



2-by-2 bits case:

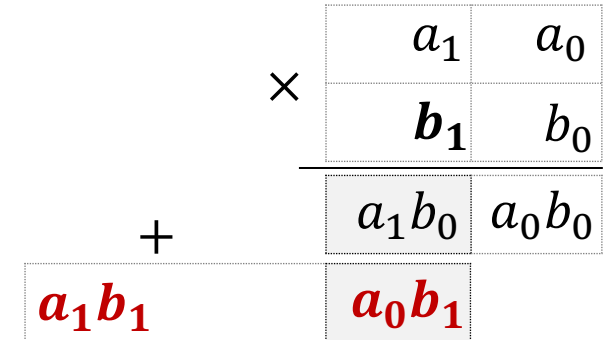
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



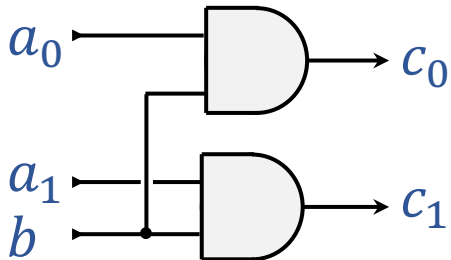
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



Binary Multipliers

1-by-1 bit case:

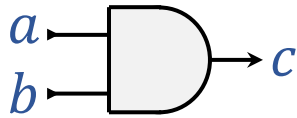
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



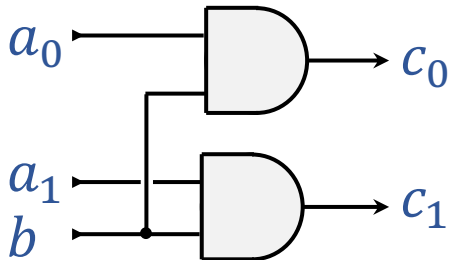
2-by-1 bits case:

$$a_1 a_0 \times b = c_1 c_0$$

$$a_1 a_0 \times 1 = a_1 a_0$$

$$a_1 a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

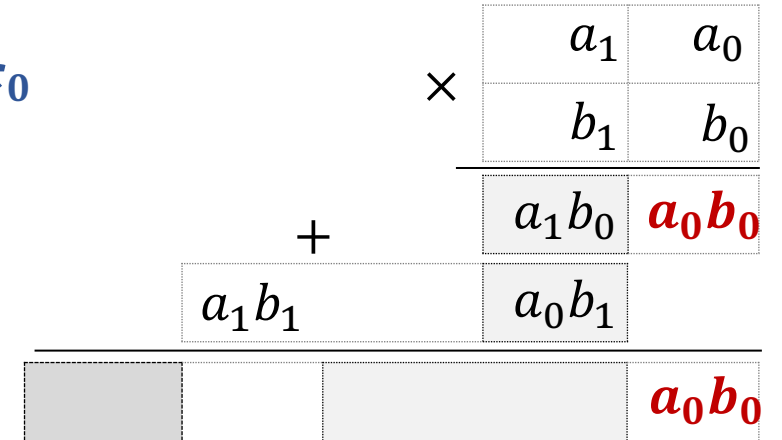
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

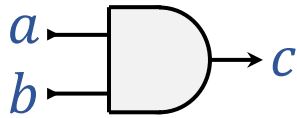
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



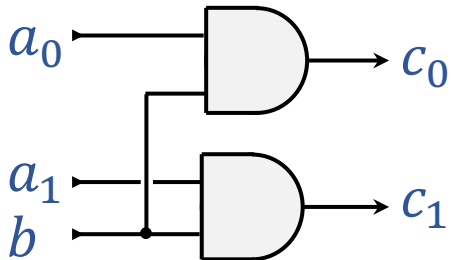
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

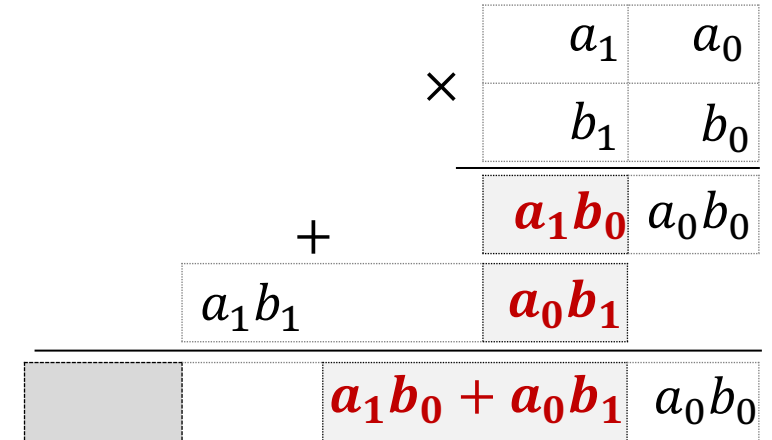
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

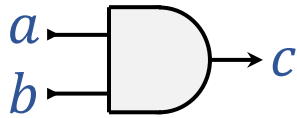
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



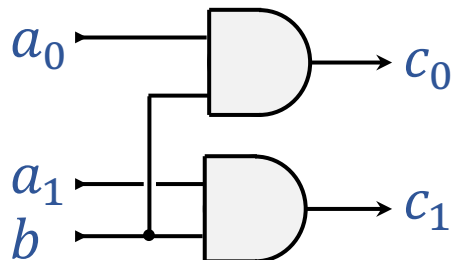
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

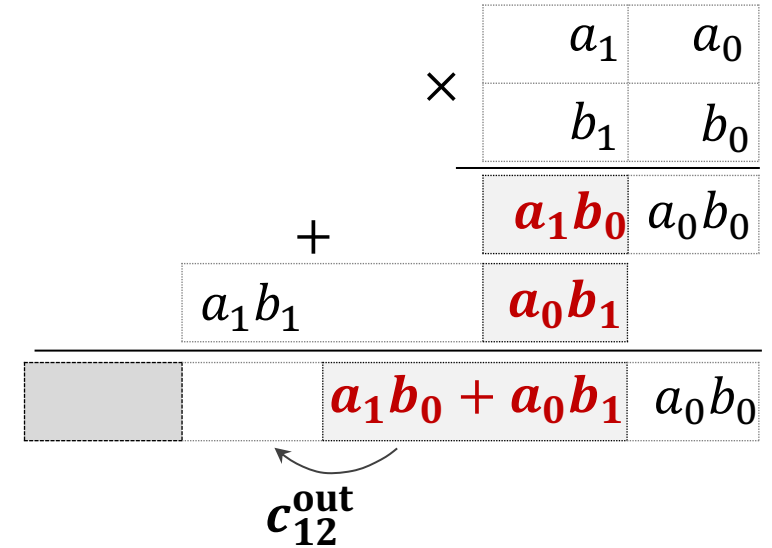
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

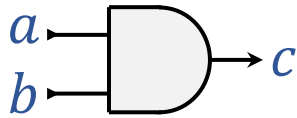
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



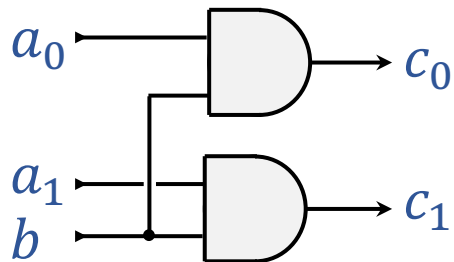
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

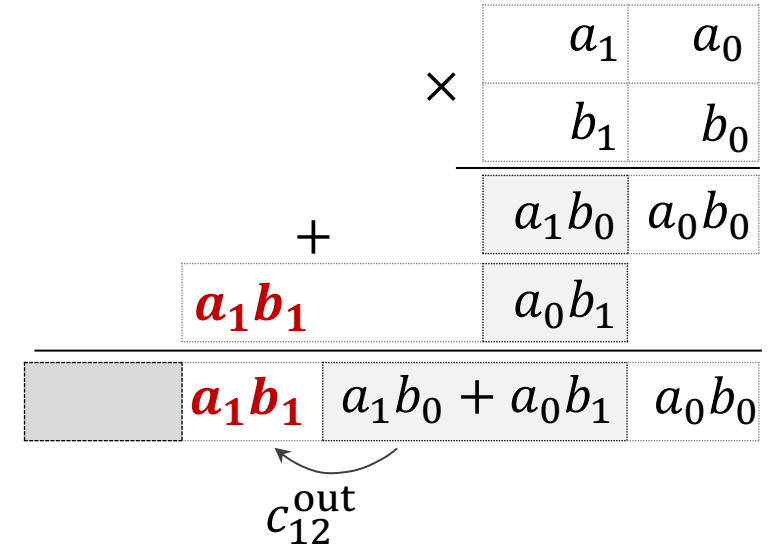
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

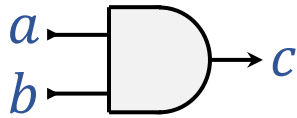
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



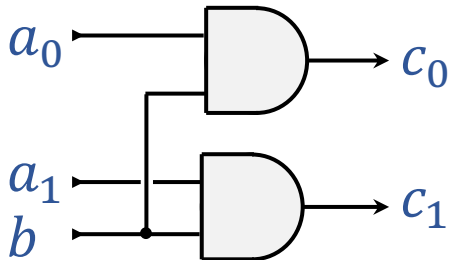
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

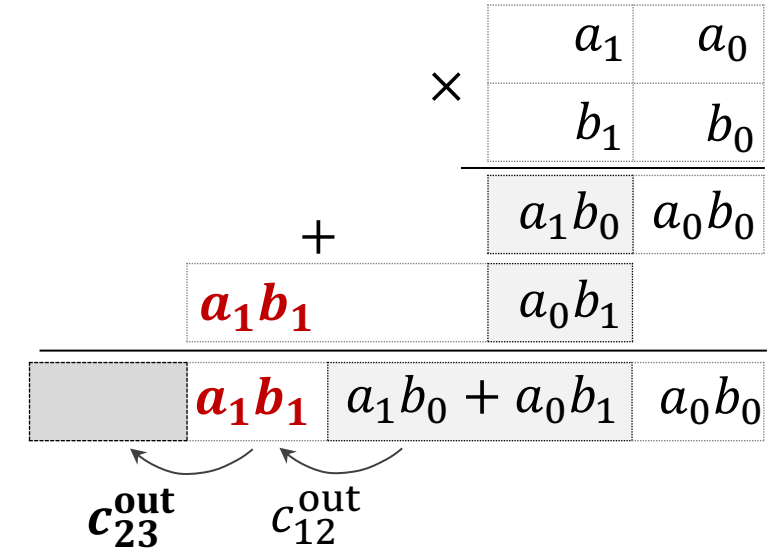
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

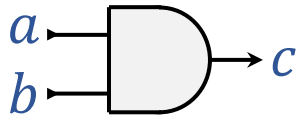
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



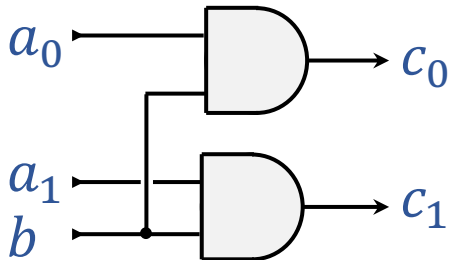
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

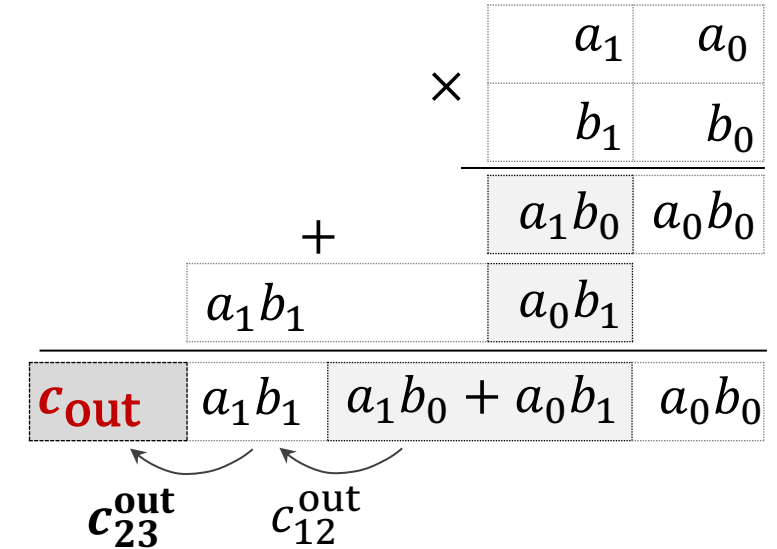
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

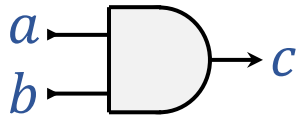
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



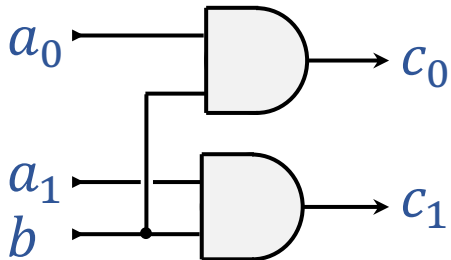
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

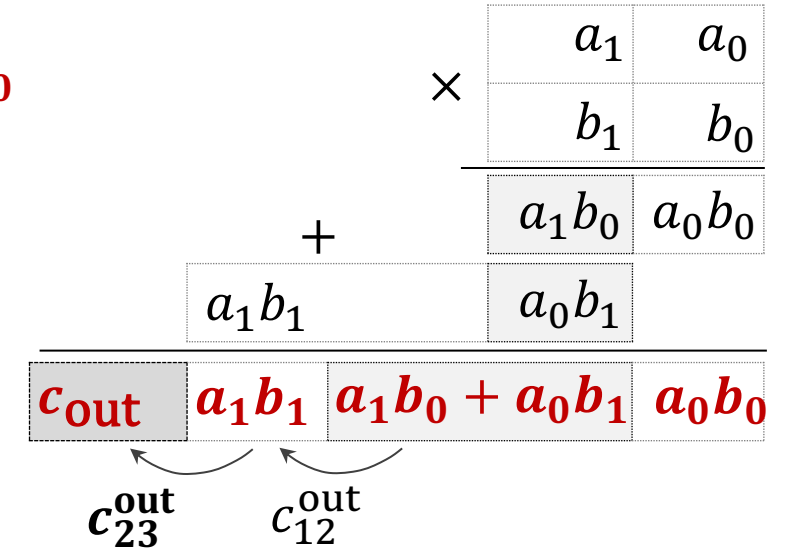
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

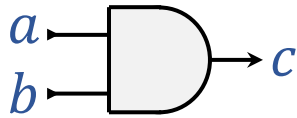
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



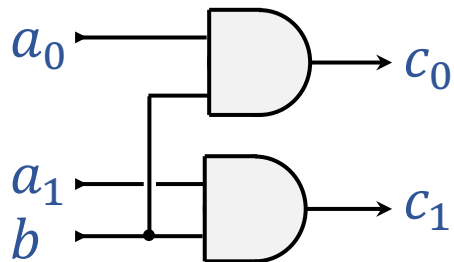
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

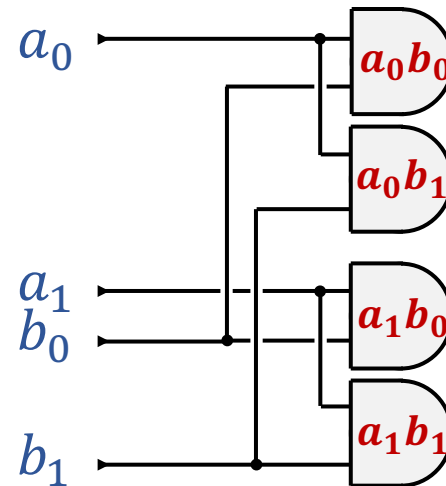
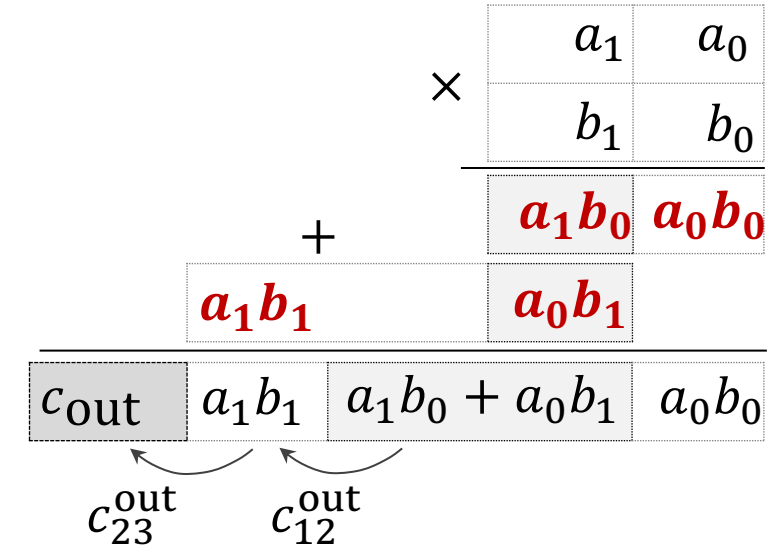
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

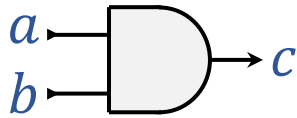
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



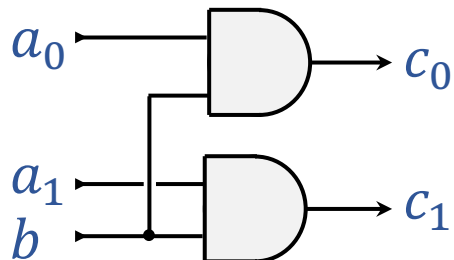
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

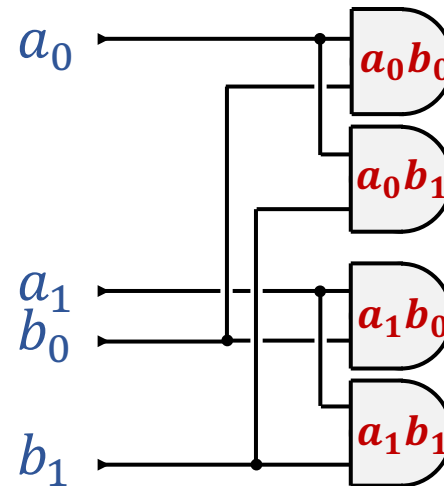
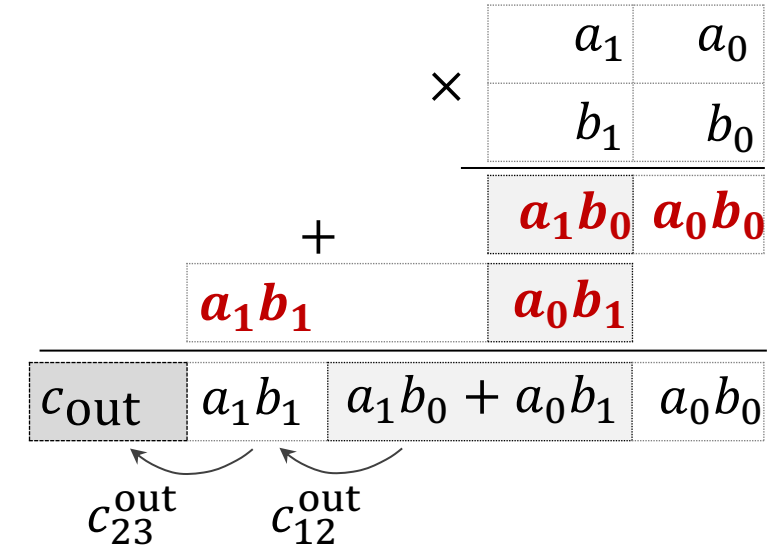
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



<- Partial products

Binary Multipliers

1-by-1 bit case:

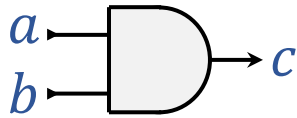
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



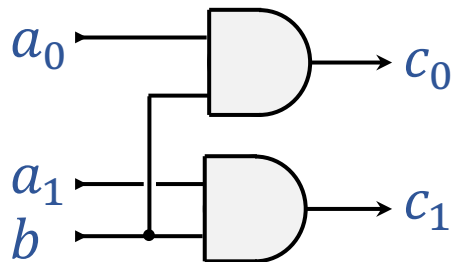
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

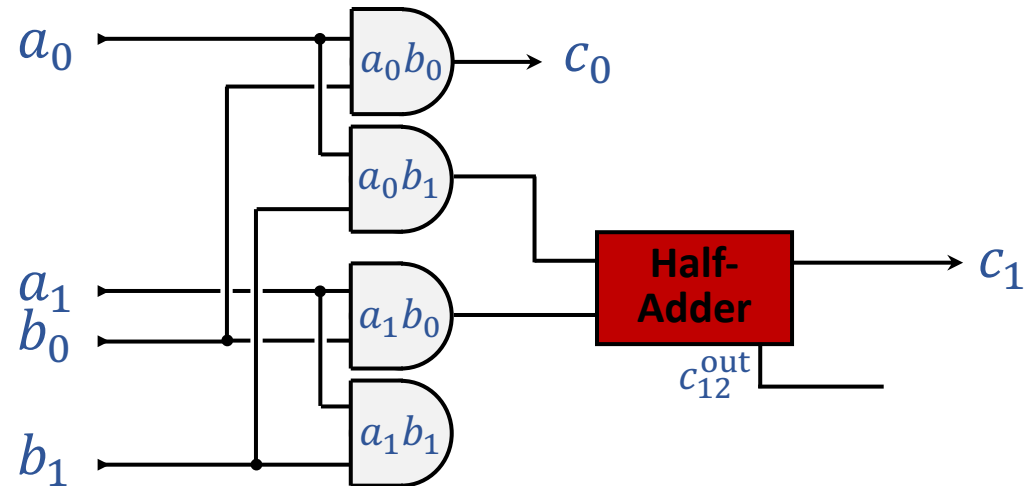
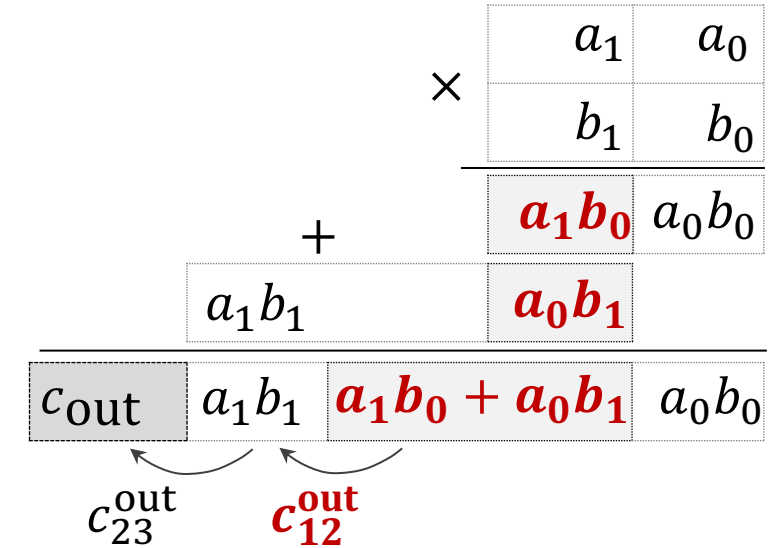
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

$$00 \times 11 = 00$$

...



Binary Multipliers

1-by-1 bit case:

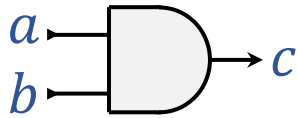
$$a \times b = c$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$



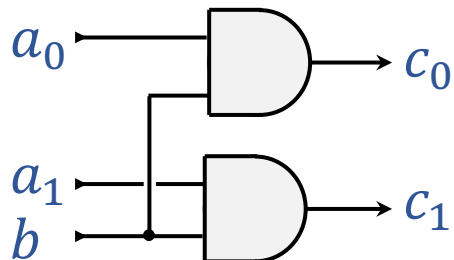
2-by-1 bits case:

$$a_1a_0 \times b = c_1c_0$$

$$a_1a_0 \times 1 = a_1a_0$$

$$a_1a_0 \times 0 = 00$$

Every bit of a_2a_1 is multiplied by b :



2-by-2 bits case:

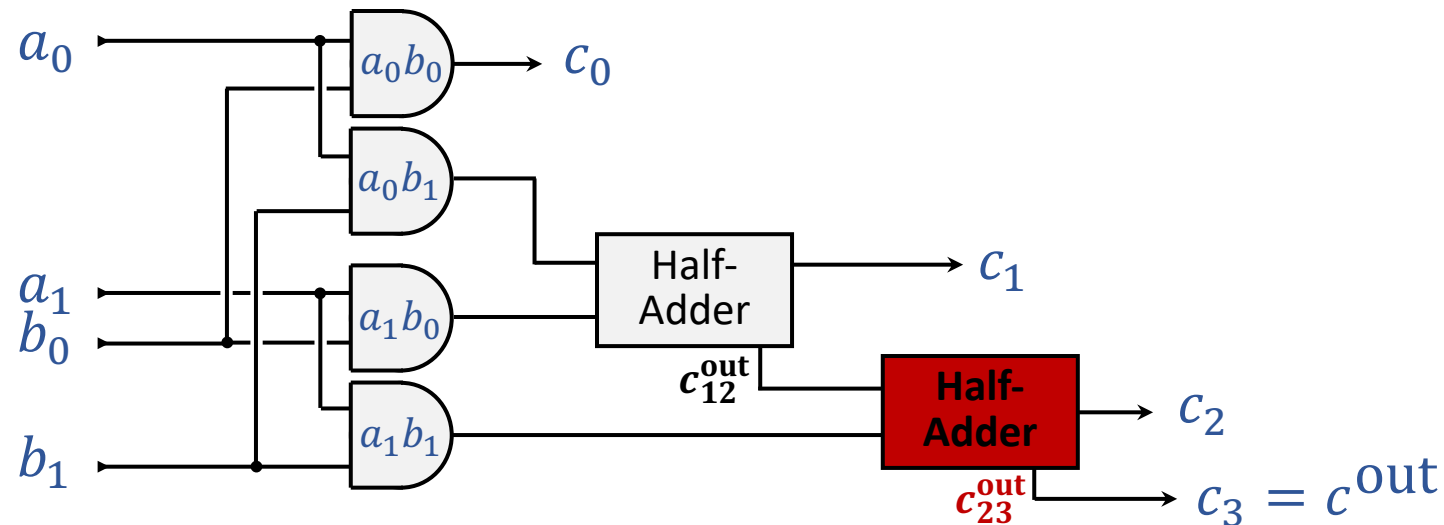
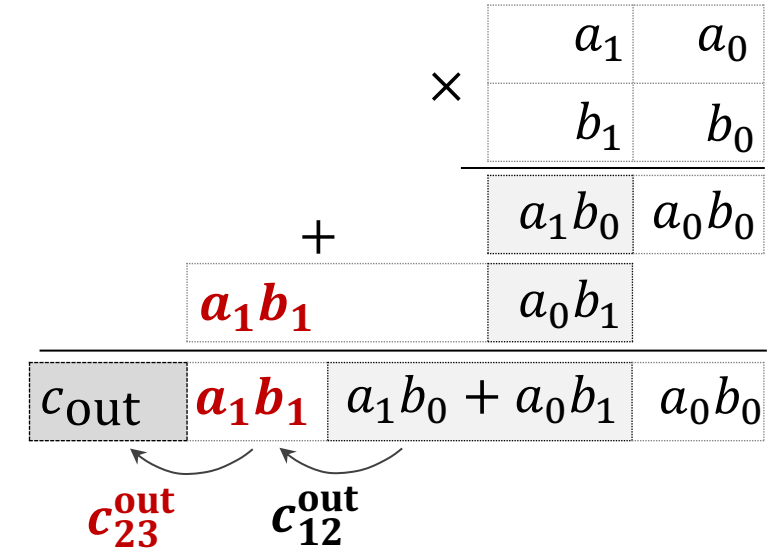
$$a_1a_0 \times b_1b_0 = c_3c_2c_1c_0$$

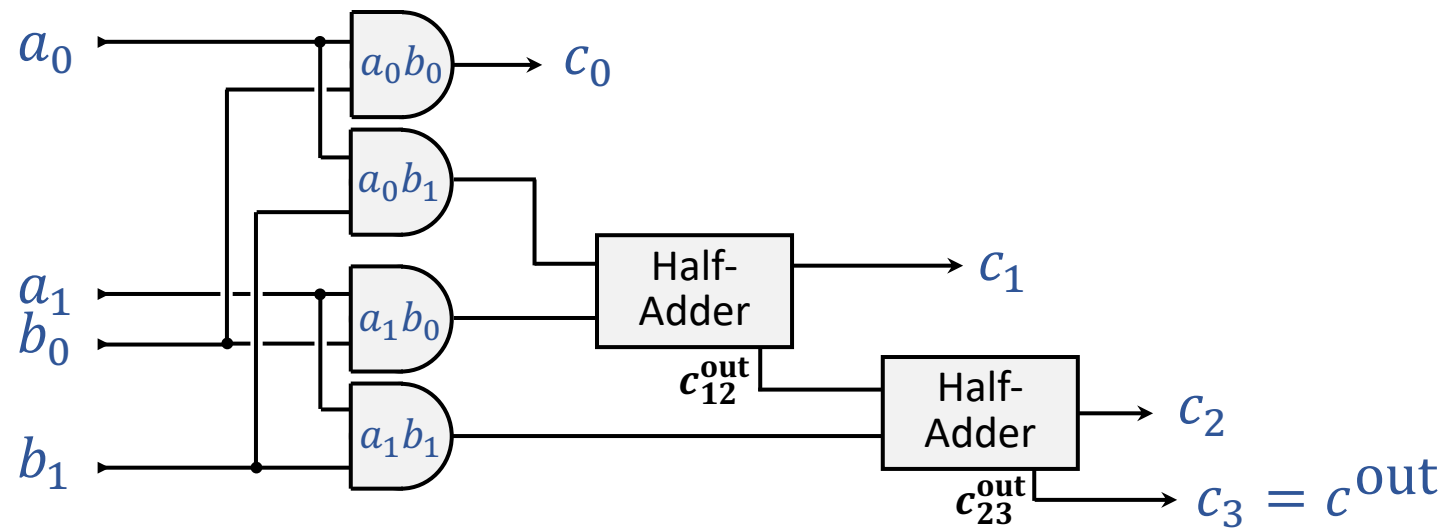
$$11 \times 11 = 1001$$

$$10 \times 01 = 010$$

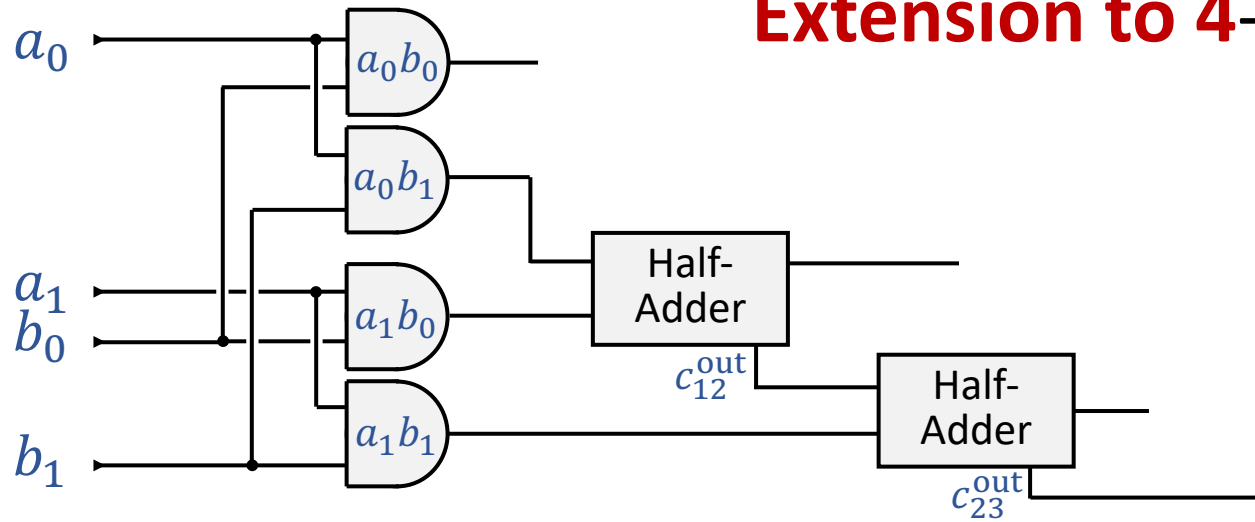
$$00 \times 11 = 00$$

...



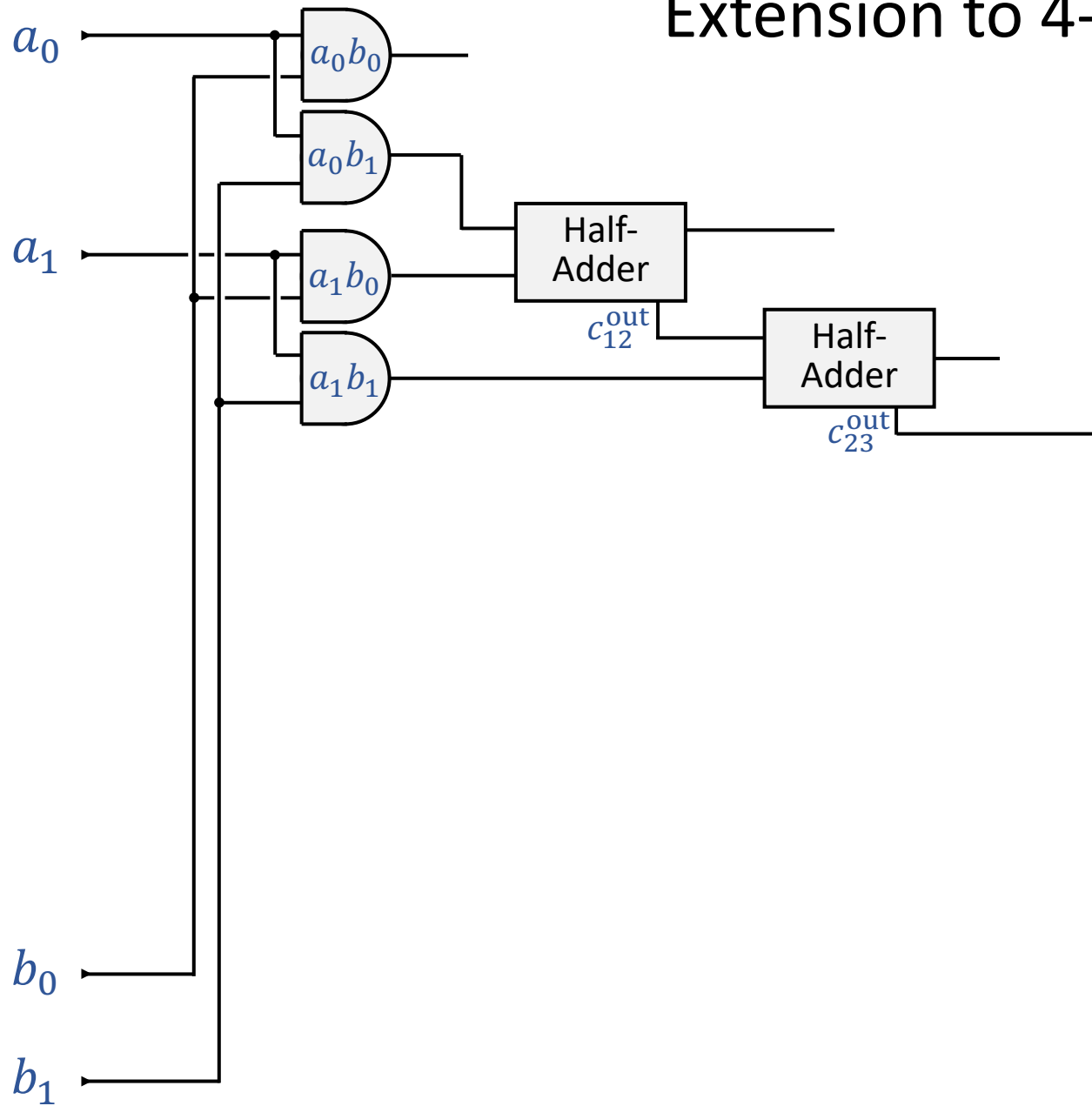


Extension to 4-by-2 Bit Multiplier Construction



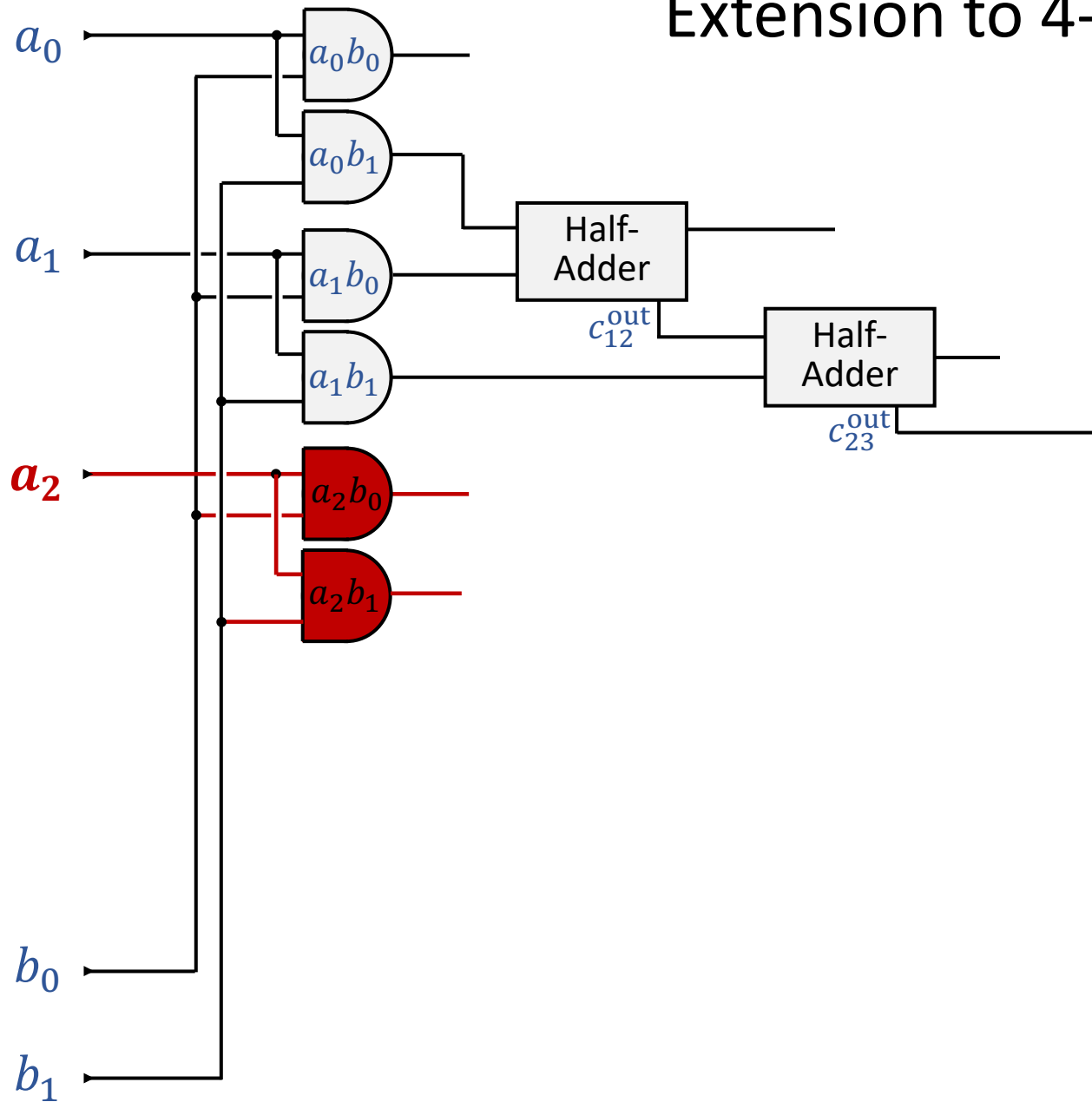
$$\begin{array}{r} \times \quad \begin{array}{|c|c|} \hline a_1 & a_0 \\ \hline b_1 & b_0 \\ \hline \end{array} \\ \hline + \quad \begin{array}{|c|c|} \hline a_1b_0 & a_0b_0 \\ \hline a_1b_1 & a_0b_1 \\ \hline \end{array} \end{array}$$

Extension to 4-by-2 Bit Multiplier Construction



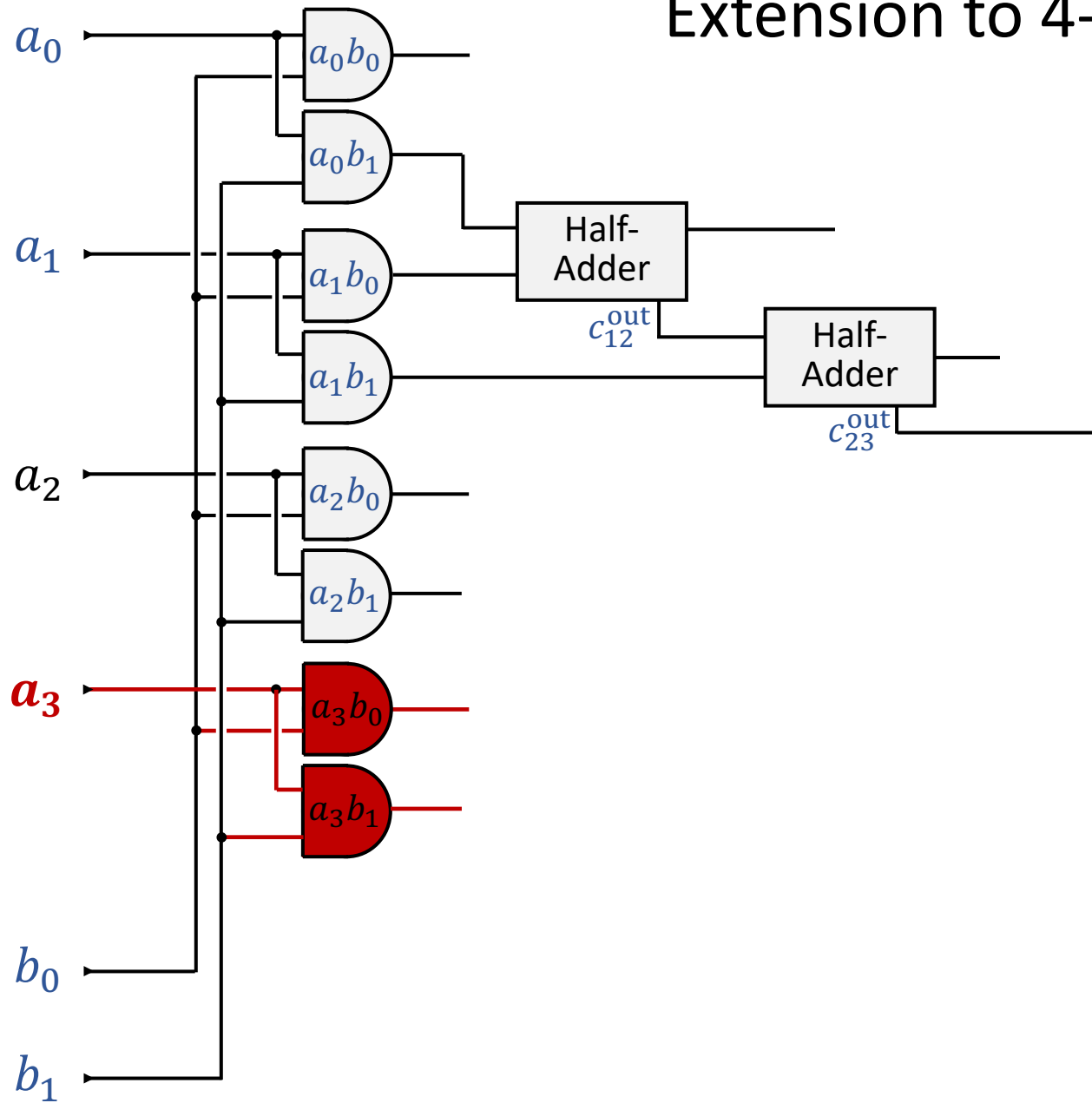
$$\begin{array}{r} \times \begin{array}{|c|c|} \hline a_1 & a_0 \\ \hline b_1 & b_0 \\ \hline \end{array} \\ + \begin{array}{|c|c|} \hline a_1b_0 & a_0b_0 \\ \hline a_1b_1 & a_0b_1 \\ \hline \end{array} \\ \hline \end{array}$$

Extension to 4-by-2 Bit Multiplier Construction



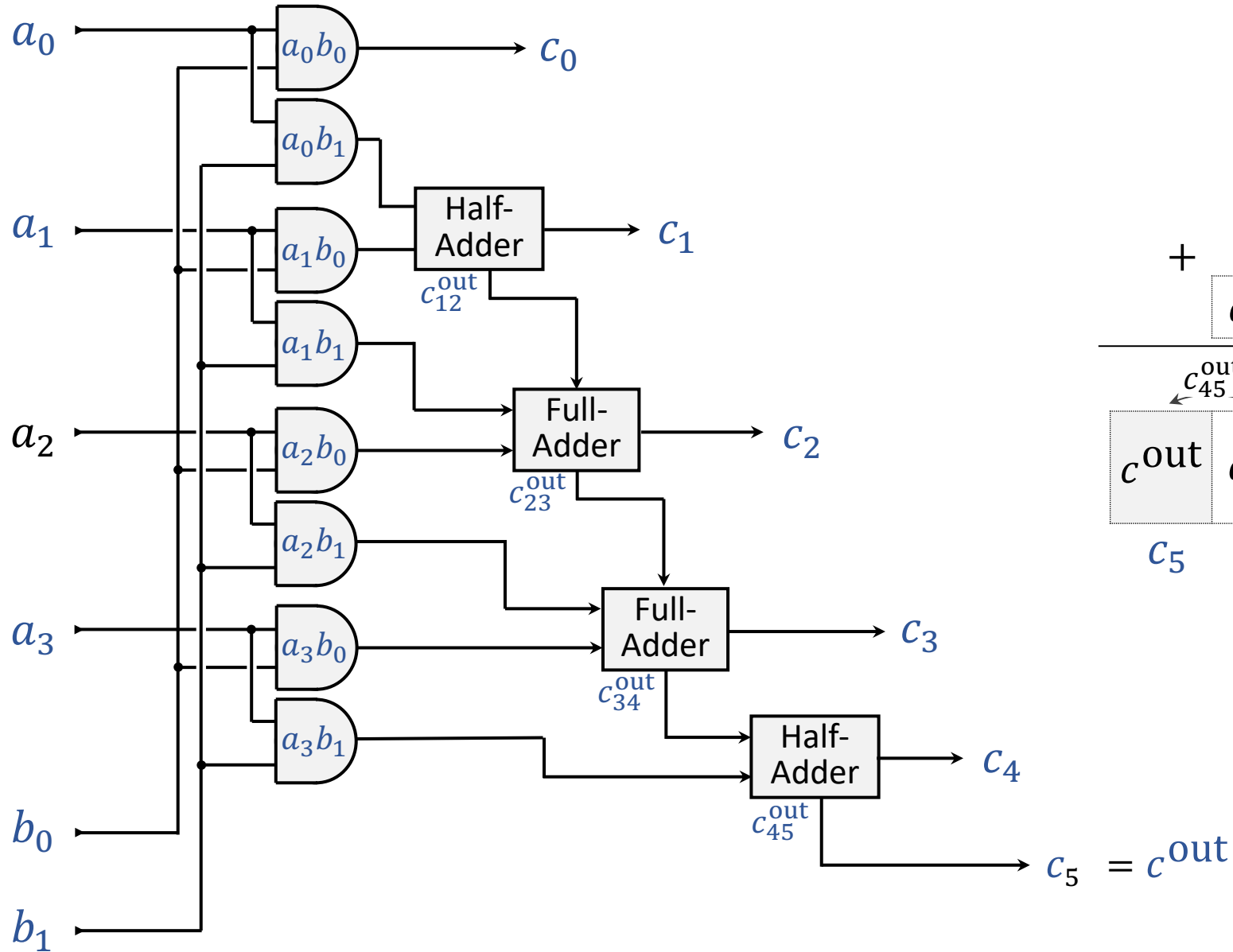
$$\begin{array}{r}
 \times \begin{array}{|c|c|c|} \hline \mathbf{a_2} & a_1 & a_0 \\ \hline & b_1 & b_0 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|} \hline \mathbf{a_2b_0} & a_1b_0 & a_0b_0 \\ \hline \mathbf{a_2b_1} & a_1b_1 & a_0b_1 \\ \hline \end{array}
 \end{array}$$

Extension to 4-by-2 Bit Multiplier Construction



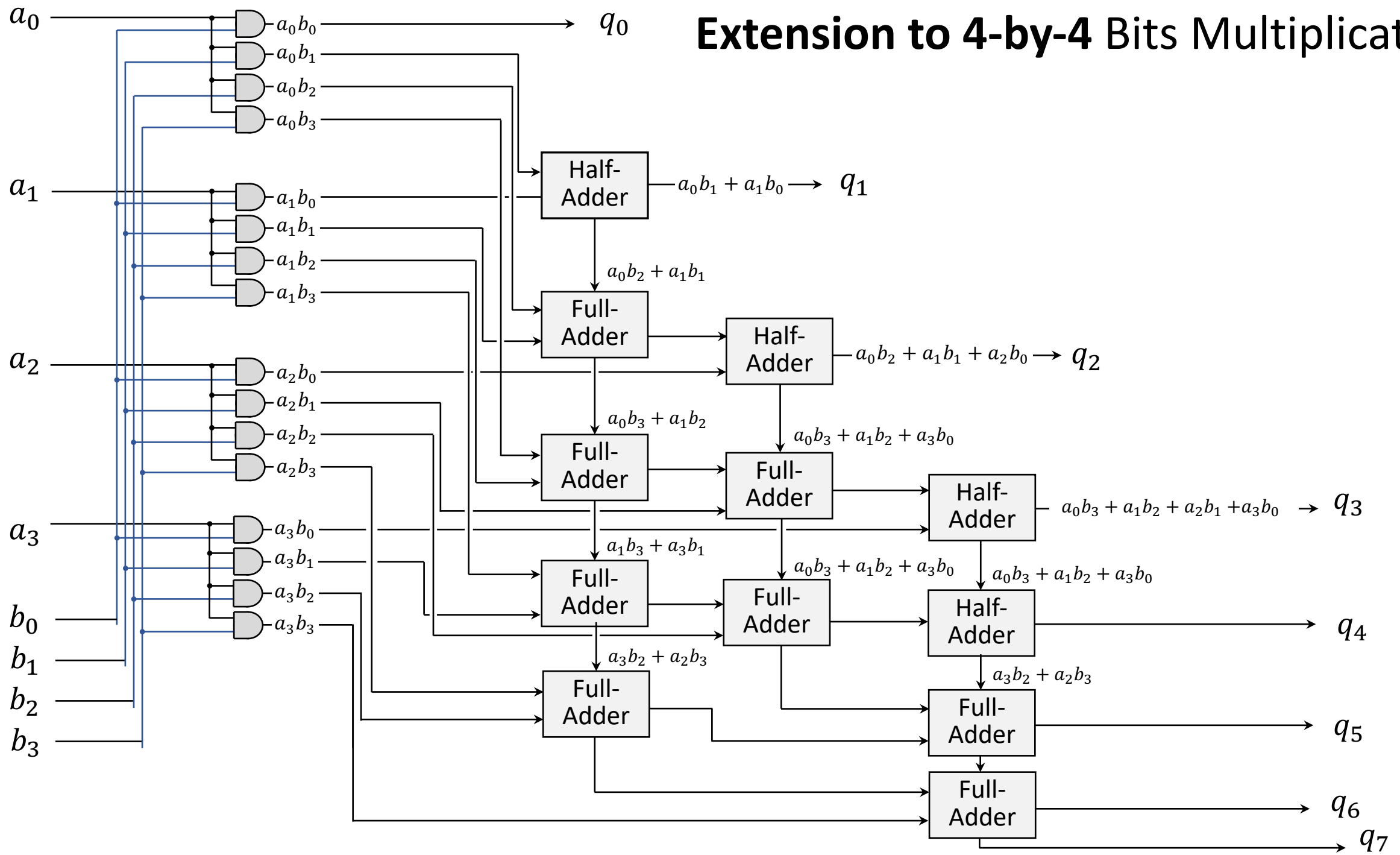
$$\begin{array}{r}
 \times \quad \begin{array}{|c|c|c|c|} \hline \mathbf{a_3} & a_2 & a_1 & a_0 \\ \hline & & b_1 & b_0 \\ \hline \end{array} \\
 + \quad \begin{array}{|c|c|c|c|} \hline \mathbf{a_3 b_0} & a_2 b_0 & a_1 b_0 & a_0 b_0 \\ \hline \mathbf{a_3 b_1} & a_2 b_1 & a_1 b_1 & a_0 b_1 \\ \hline \end{array}
 \end{array}$$

4-by-2 Bit Multiplier Construction

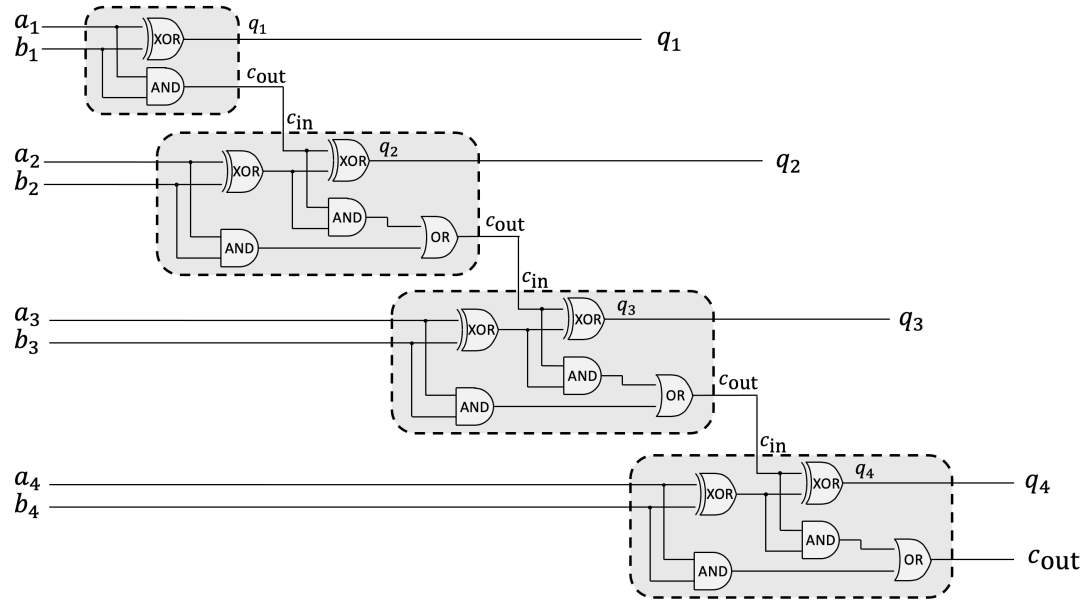


$$\begin{array}{r}
 \times \begin{array}{|c|c|c|c|} \hline a_3 & a_2 & a_1 & a_0 \\ \hline & & b_1 & b_0 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline a_3 b_0 & a_2 b_0 & a_1 b_0 & a_0 b_0 \\ \hline a_3 b_1 & a_2 b_1 & a_1 b_1 & a_0 b_1 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|c|c|} \hline c^{\text{out}} & a_3 b_1 & a_2 b_1 + a_3 b_0 & a_1 b_1 + a_2 b_0 & a_0 b_1 + a_1 b_0 & a_0 b_0 \\ \hline \end{array} \\
 \begin{array}{cccccc}
 \leftarrow c_{45}^{\text{out}} & \leftarrow c_{34}^{\text{out}} & \leftarrow c_{23}^{\text{out}} & \leftarrow c_{12}^{\text{out}} & & \\
 c_5 & c_4 & c_3 & c_2 & c_1 & c_0
 \end{array}
 \end{array}$$

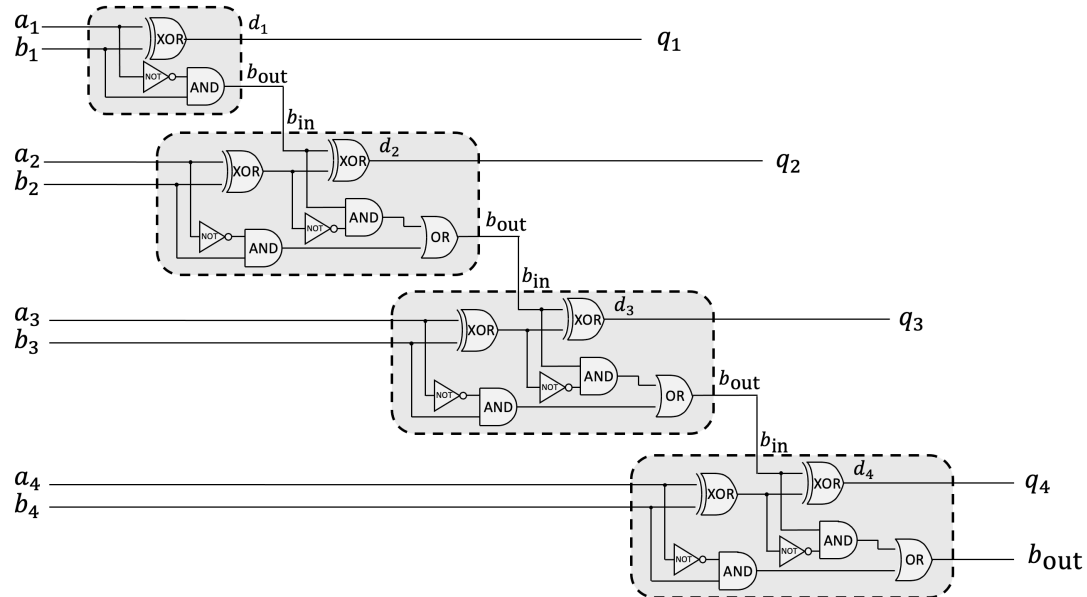
Extension to 4-by-4 Bits Multiplier



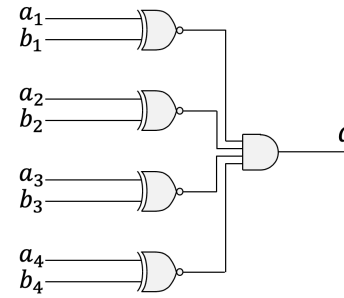
Adder (+)



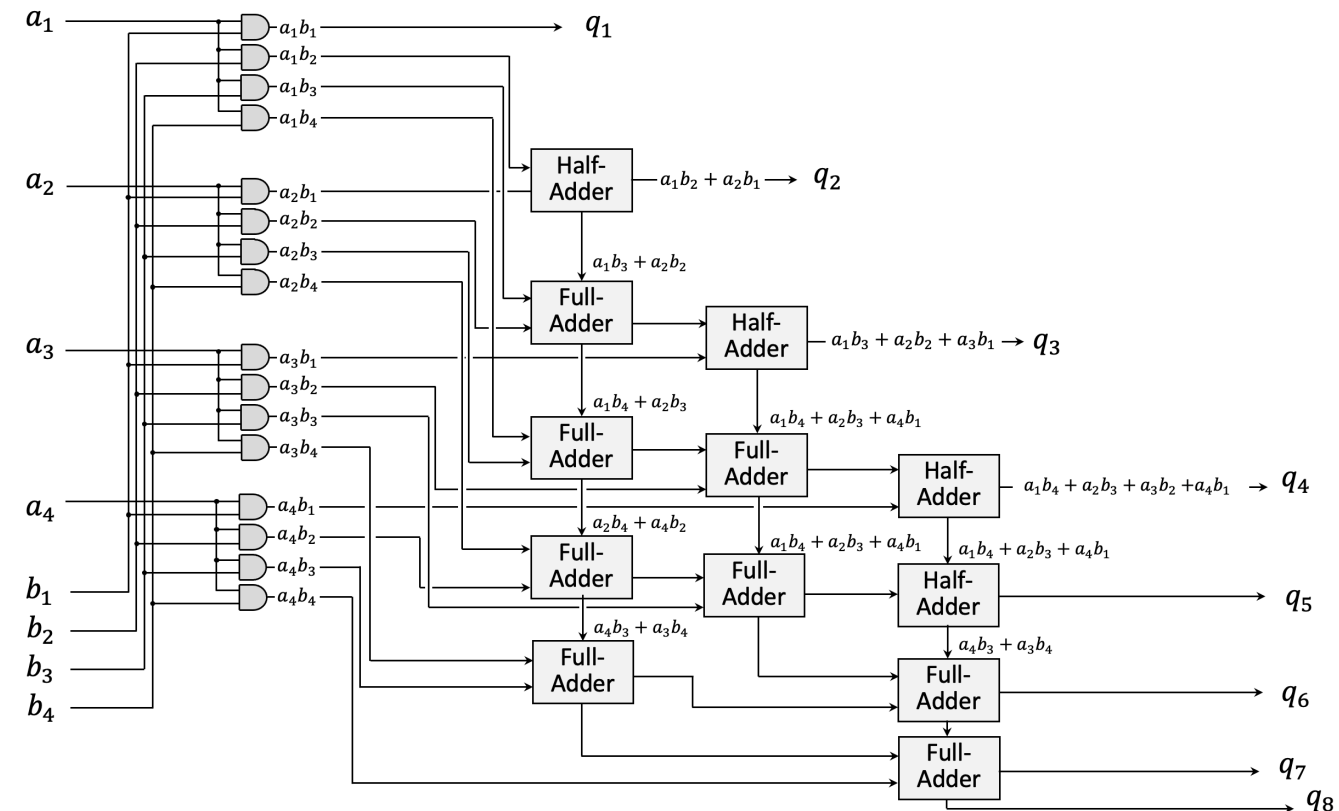
Subtractor (-)



Equality Checker (=?)



Multiplicator (*)



Some Instructions
Implementations